

Systems of two heavy quarks with Effective Field Theories

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Quarkonium Working Group

Heavy Quarkonium play a role in so many high energy processes:

- $c\bar{c}$ [BES](#), [BES-III](#), [E835](#), [KEDR](#), [CLEO-c](#); $b\bar{b}$ [CLEO-III](#)
- Production at ([Fermilab \(CDF, D0\)](#))
- Production at ([Hera \(Zeus, H1\)](#))
- Production at ([B factories \(BaBar, Belle\)](#))
- Quark-gluon plasma ([NA60](#) at CERN, Star and Phenix at RHIC)
- Physics at [ILC](#)
- LHC at CERN, Panda at GSI

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- accessible to QCD studies.
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- systems where low energy QCD may be studied in a systematic way (e.g. non-perturbative matrix elements, QCD vacuum, confinement, exotica, ...).
- allow to study the Quantum Mechanics of a non-Abelian Field Theory.

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- The hierarchy of energy scales in quarkonium allows the contructions of effective field theories with less and less degrees of freedom. This leads to a field theory derived **quantum mechanical** description of these systems: pNRQCD.

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- weakly coupled pNRQCD (systems with small r).

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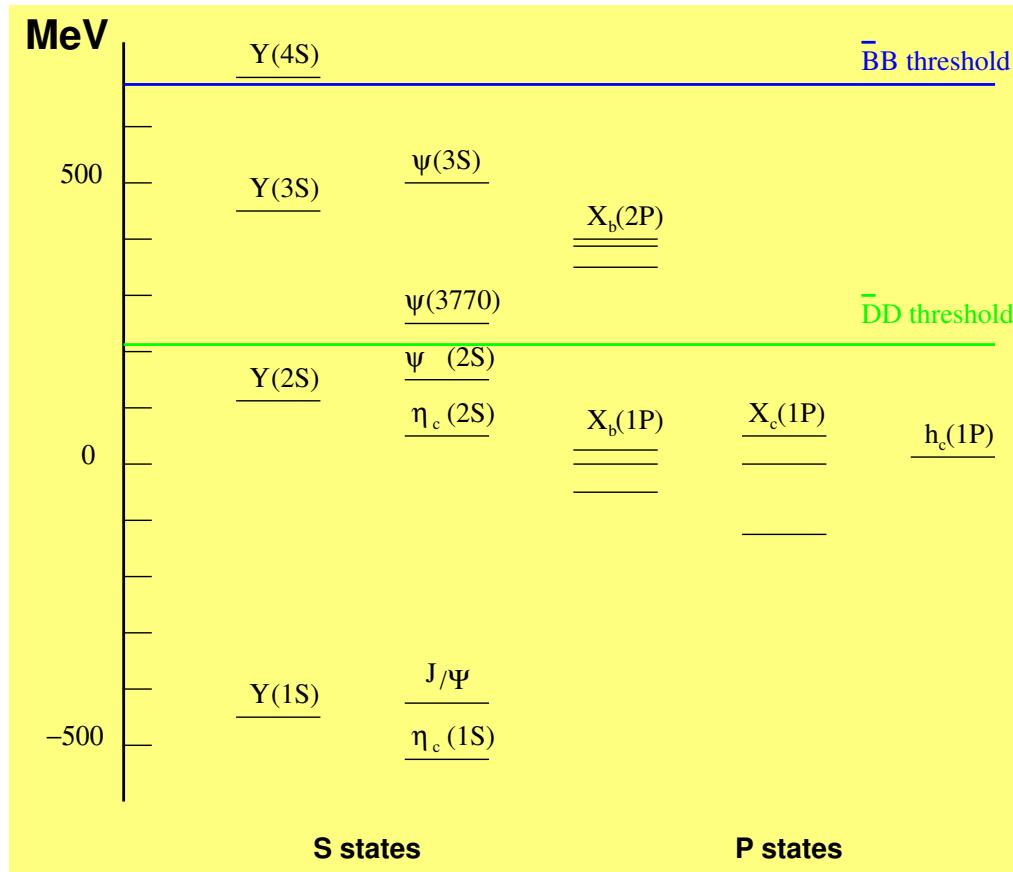
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- weakly coupled pNRQCD (systems with small r).
- strongly coupled pNRQCD (systems with r larger than the confinement scale)
- Applications: Potentials, Spectra, Annihilations, Transitions, Gluelumps

Scales and EFTs

Quarkonium Scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

The mass scale is perturbative:

$$m_b \simeq 5 \text{ GeV}, m_c \simeq 1.5 \text{ GeV}$$

The system is non-relativistic:

$$\Delta_n E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \simeq 0.1, v_c^2 \simeq 0.3$$

Non-relativistic bound states are characterized by at least three energy scales

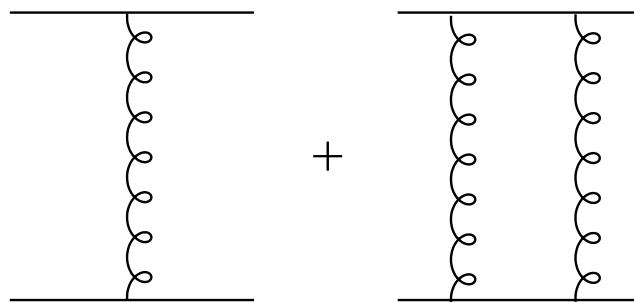
$$m \gg m\textcolor{blue}{v} \gg m\textcolor{blue}{v}^2 \quad v \ll 1$$

Quarkonium Scales

Another small parameter shows up near threshold:

$$E \approx 2m + \frac{p^2}{m} + \dots \quad \text{with} \quad v = \frac{p}{m} \ll 1$$

- The perturbative expansion breaks down when $\alpha_s \sim v$:



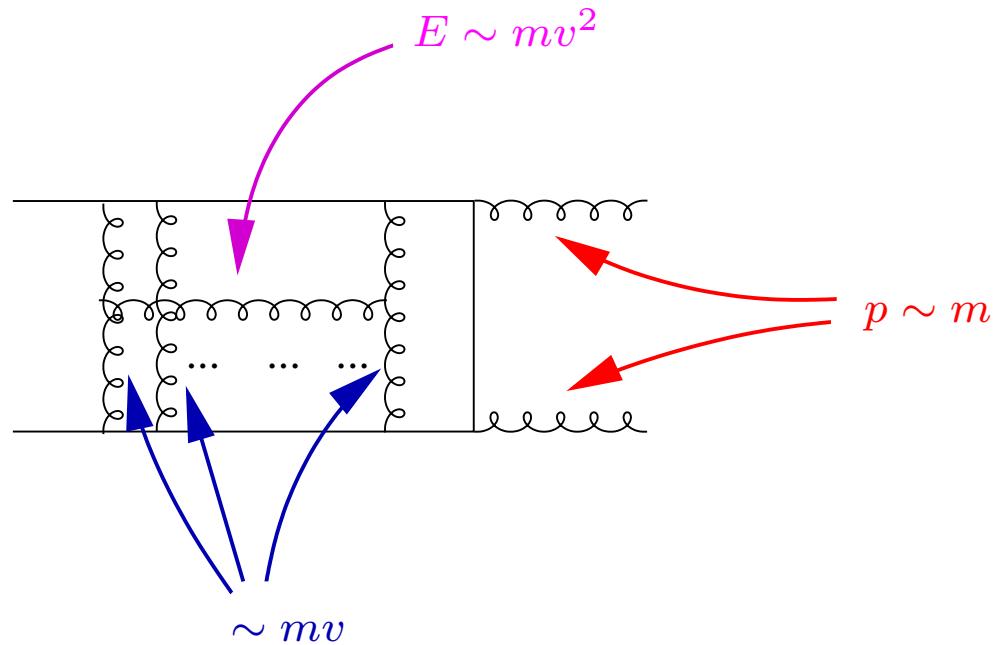
The diagram shows a series of Feynman diagrams representing a perturbative expansion. It consists of two horizontal lines connected by a vertical gluon loop. The first diagram is followed by a plus sign, then a second diagram with two loops, another plus sign, and three dots indicating higher-order terms. This visualizes the expansion $E \approx 2m + \frac{p^2}{m} + \dots$.

$$\approx \frac{1}{E - \left(\frac{p^2}{m} + V \right)}$$
$$\alpha_s \left(1 + \frac{\alpha_s}{v} + \dots \right)$$

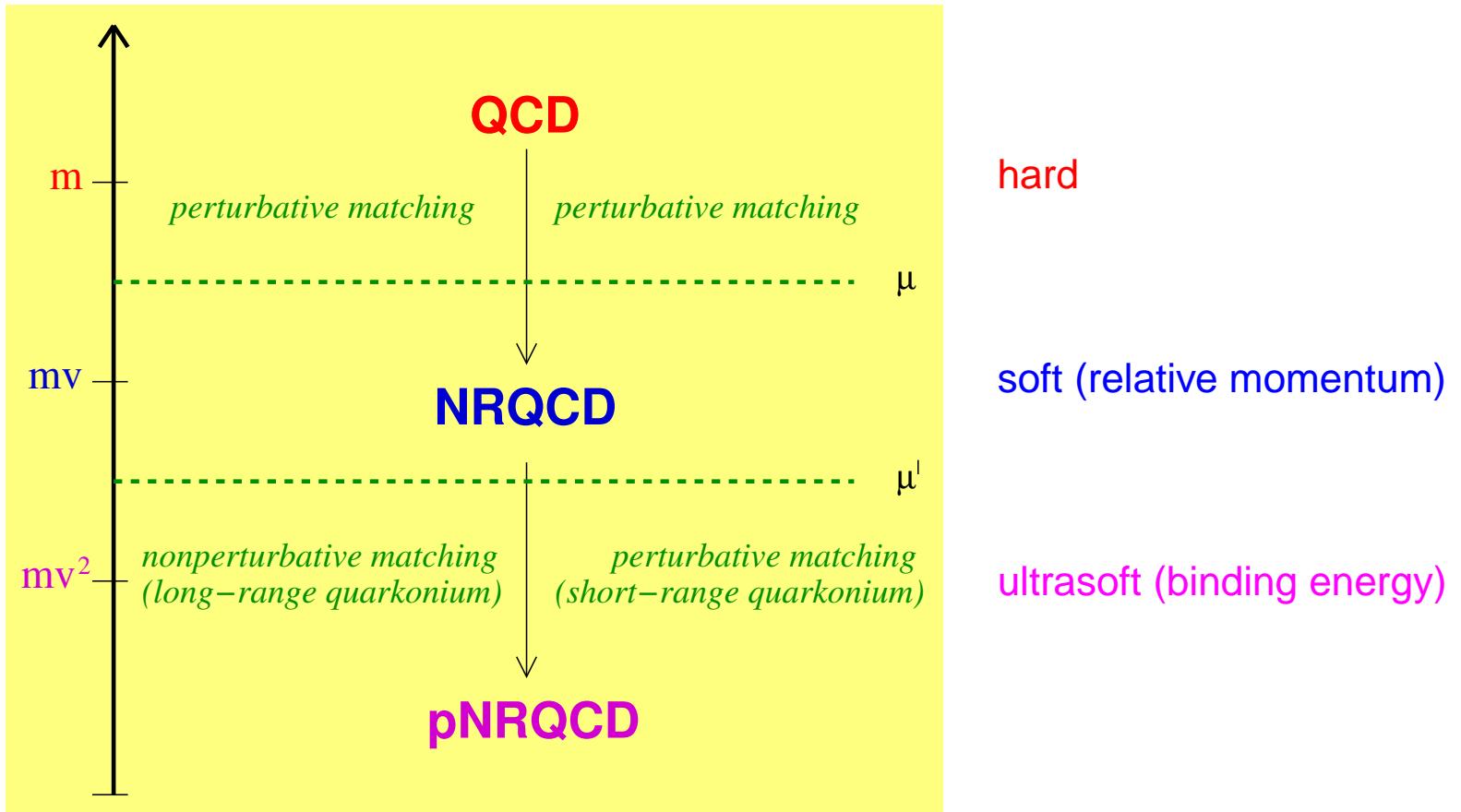
- The system is non-relativistic : $p \sim m v$ and $E = \frac{p^2}{m} + V \sim m v^2$.

Quarkonium Scales

Scales get entangled.



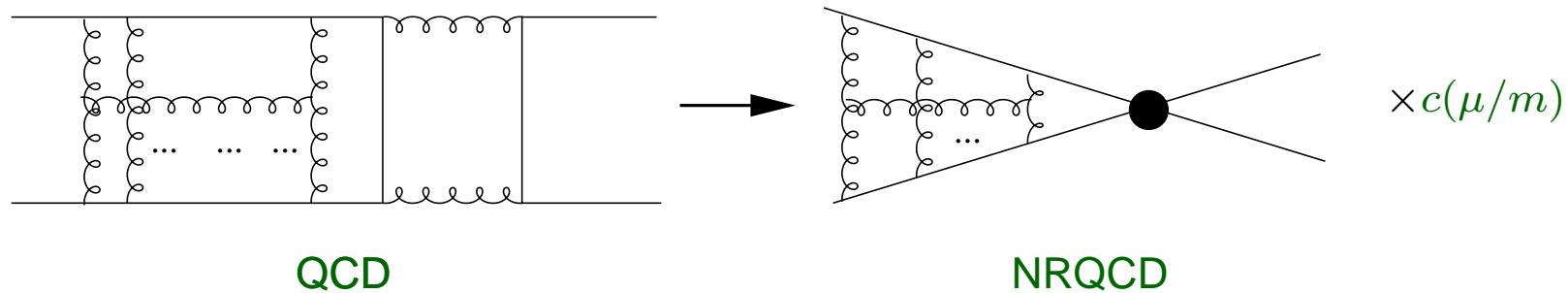
Effective Field Theories for quarkonium



In QCD another scale is relevant: Λ_{QCD}

NRQCD

NRQCD is the EFT that follows from QCD when $\Lambda = m$



- The matching is perturbative.
- The Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

$$\mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times O_n(\mu, \lambda) / m^n$$

Suitable to describe decay and production of quarkonium.

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \textcolor{blue}{c_F} \frac{\mathbf{S} \cdot g\mathbf{B}}{m} + \textcolor{blue}{c_D} \frac{[\mathbf{D} \cdot, g\mathbf{E}]}{8m^2} + \dots \right) \psi$$

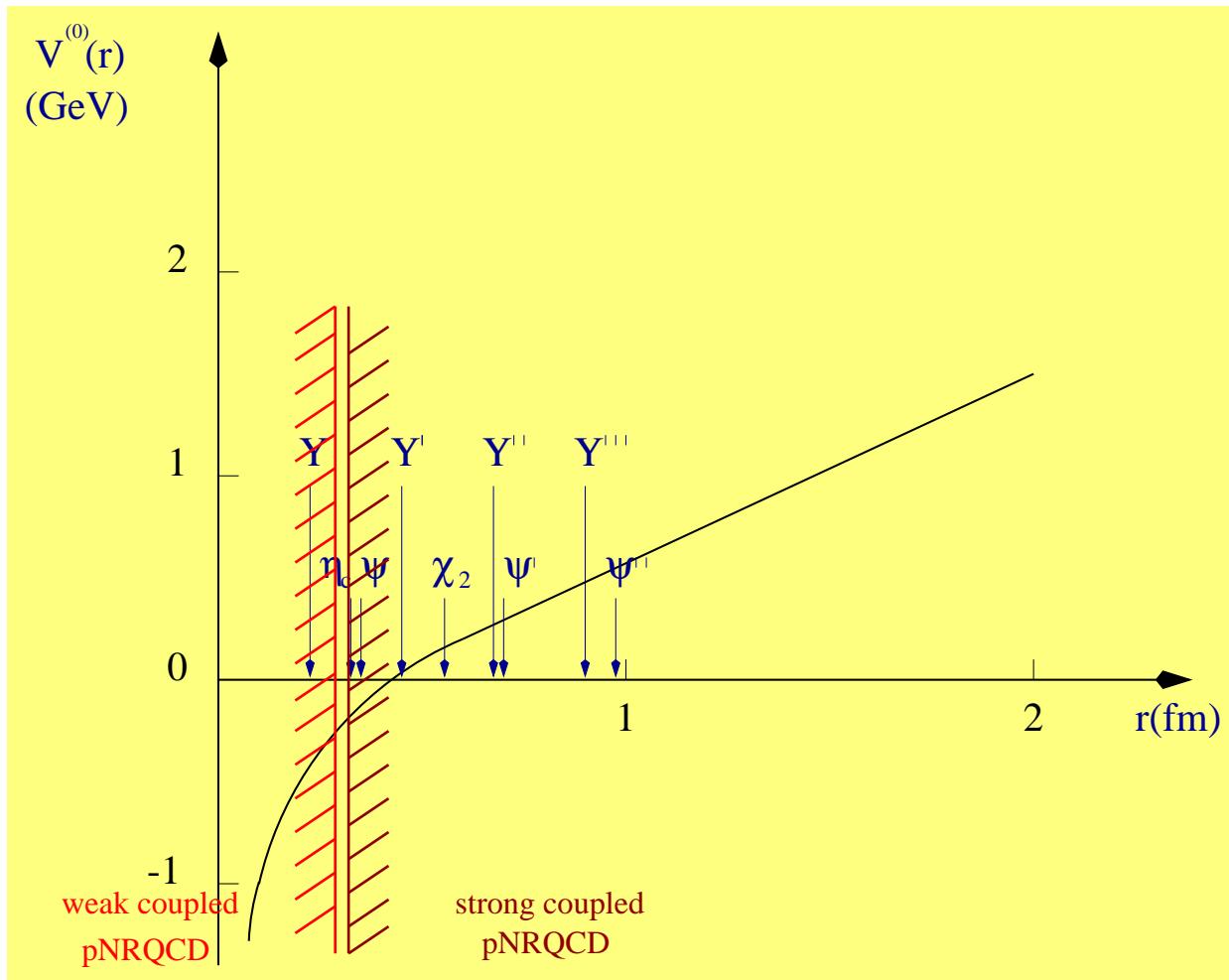
$1 + (\cdot \chi^\dagger) \left(\textcolor{blue}{\alpha_S} \mathbb{I} + \dots - \frac{\mathbf{D}^2}{2m} - c_F \frac{\mathbf{S} \cdot g\mathbf{B}}{m} - c_D \frac{[\mathbf{D} \cdot, g\mathbf{E}]}{8m^2} + \dots \right) \chi$

$f = \text{Re } f + \textcolor{red}{i} \text{Im } f$

$$+ \sum_K \frac{\textcolor{blue}{f}}{m^2} \psi^\dagger \ K \ \chi \chi^\dagger \ K \ \psi + \dots$$

$$- \frac{1}{4} F_{\mu\nu}^a F^{a\ \mu\nu} + \sum_{q=1}^{n_f} \bar{q} \ i \not{D} q + \dots$$

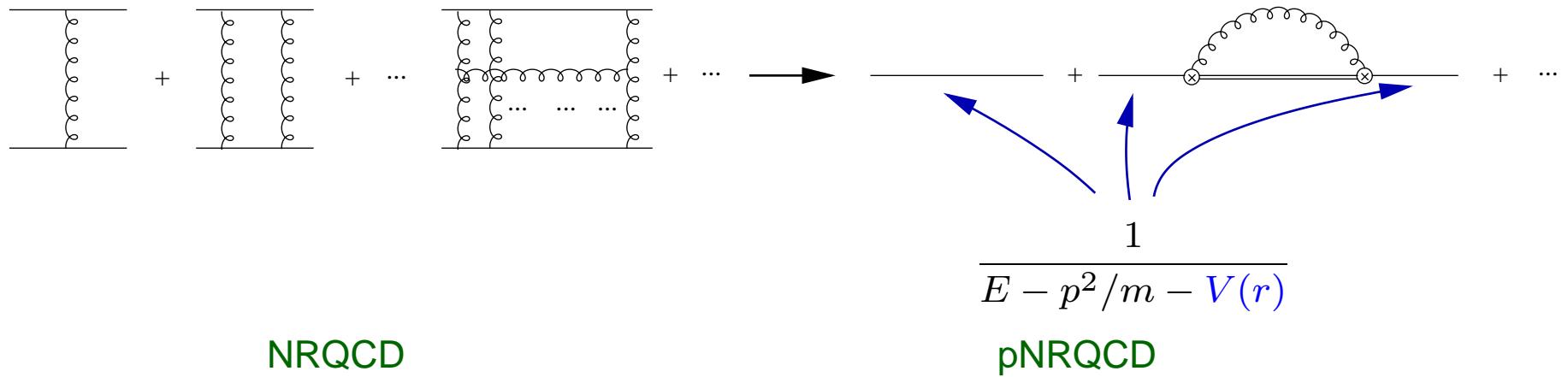
The scale $r \sim 1/mv$



Weakly Coupled pNRQCD

pNRQCD for $Q\bar{Q}$

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = \frac{1}{r} \sim mv$



- The Lagrangian is organized as an expansion in $1/m$, r , and $\alpha_s(m)$:

$$\mathcal{L}_{\text{pNRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda) r^n$$

pNRQCD Lagrangian for $Q\bar{Q}$

- If $mv \gg \Lambda_{\text{QCD}}$, the matching is perturbative
- Degrees of freedom: quarks and gluons

$Q\bar{Q}$ states, with energy $\sim \Lambda_{\text{QCD}}$, mv^2 and momentum $\lesssim mv$
 \Rightarrow (i) singlet S (ii) octet O

Gluons with energy and momentum $\sim \Lambda_{\text{QCD}}, mv^2$

- Definite power counting: $r \sim \frac{1}{mv}$ and $t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\text{QCD}}}$

The gauge fields are multipole expanded:

$$A(R, r, t) = A(R, t) + \mathbf{r} \cdot \nabla A(R, t) + \dots$$

Non-analytic behaviour in $r \rightarrow$ matching coefficients V

pNRQCD Lagrangian for $Q\bar{Q}$

$$\mathcal{L}_{\text{pNRQCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \text{Tr} \left\{ \textcolor{magenta}{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - \textcolor{blue}{V}_s \right) \textcolor{magenta}{S} \right.$$

$$\left. + \textcolor{magenta}{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \textcolor{magenta}{O} \right\}$$

LO in $\textcolor{green}{r}$

$$+ \textcolor{blue}{V}_A \text{Tr} \left\{ \textcolor{magenta}{O}^\dagger \mathbf{r} \cdot g \mathbf{E} \textcolor{magenta}{S} + \textcolor{magenta}{S}^\dagger \mathbf{r} \cdot g \mathbf{E} \textcolor{magenta}{O} \right\}$$

$$+ \frac{\textcolor{blue}{V}_B}{2} \text{Tr} \left\{ \textcolor{magenta}{O}^\dagger \mathbf{r} \cdot g \mathbf{E} \textcolor{magenta}{O} + \textcolor{magenta}{O}^\dagger \textcolor{magenta}{O} \mathbf{r} \cdot g \mathbf{E} \right\}$$

$$+ \dots$$

NLO in $\textcolor{green}{r}$

Pineda Soto 97, Brambilla Pineda Soto Vairo 99, 00, 03

$Q\bar{Q}$ Potential

$$V = \left(\text{---} \begin{array}{c} | \\ \text{---} \end{array} + \text{---} \begin{array}{c} | \\ \text{---} \end{array} + \dots + \begin{array}{c} | \\ \text{---} \end{array} \end{array} \end{array} \end{array} + \dots \right) - \text{---} \begin{array}{c} | \\ \text{---} \end{array} \end{array} \end{array} \end{array} \end{array} + \dots$$

$$= -\frac{4 \alpha_s}{3 r} \left[1 + a_1 \alpha_s(r) + a_2 (\alpha_s(r))^2 + \frac{9}{4} \frac{\alpha_s^3}{\pi} \ln \mu r + \dots \right]$$

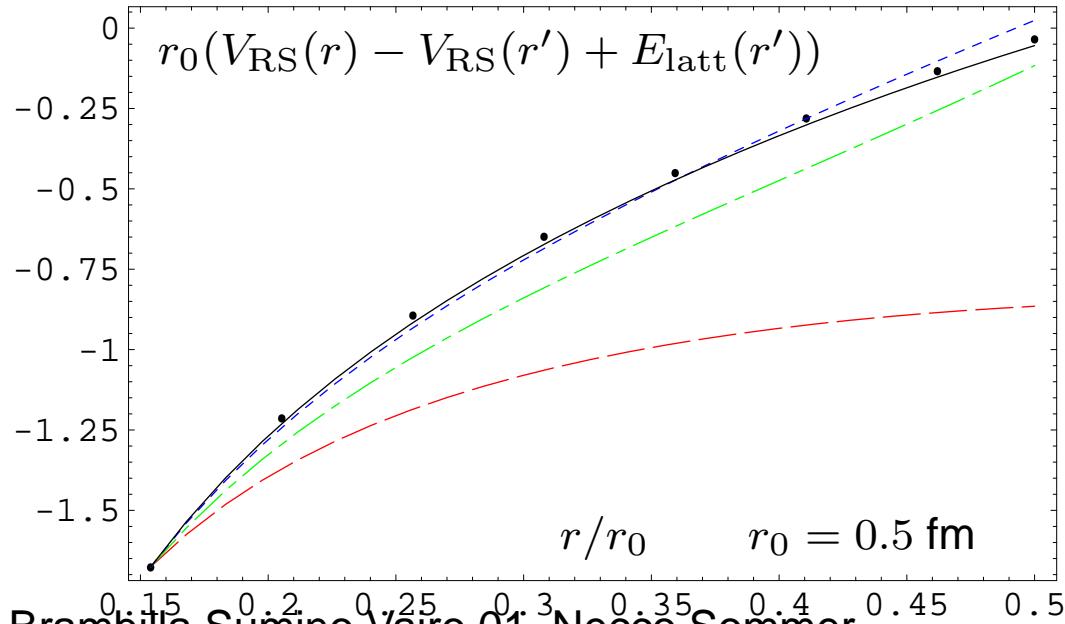
Brambilla Pineda Soto Vairo 99

$Q\bar{Q}$ Potential

$$V = \left(\begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ + \dots \\ \text{Diagram n} \\ + \dots \end{array} \right) - \text{Diagram with a loop} + \dots$$

$= -\frac{4}{3} \frac{\alpha_s}{r} \left[1 + a_1 \alpha_s(r) + a_2 (\alpha_s(r))^2 + \frac{9}{4} \frac{\alpha_s^3}{\pi} \ln \mu r + \dots \right]$

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pNRQCD for QQq

- The (weakly coupled) EFT for QQq baryons contains:

$(QQ)_{\bar{3}} = (T^1, T^2, T^3)$, $(QQ)_6 = (\Sigma^1, \dots, \Sigma^6)$, q and gluons.

$$Q_1{}_i(\mathbf{x}_1)Q_2{}_j(\mathbf{x}_2) \sim \sum_{\ell=1}^3 T^\ell(\mathbf{r}, \mathbf{R}) \underline{\mathbf{T}}_{ij}^\ell + \sum_{\sigma=1}^6 \Sigma^\sigma(\mathbf{r}, \mathbf{R}) \underline{\mathbf{\Sigma}}_{ij}^\sigma \quad i, j = 1, 2, 3$$

$$\underline{\mathbf{T}}_{ij}^\ell = \frac{1}{\sqrt{2}} \epsilon_{\ell ij},$$

$$\underline{\mathbf{\Sigma}}_{11}^1 = \underline{\mathbf{\Sigma}}_{22}^4 = \underline{\mathbf{\Sigma}}_{33}^6 = 1,$$

$$\underline{\mathbf{\Sigma}}_{12}^2 = \underline{\mathbf{\Sigma}}_{21}^2 = \underline{\mathbf{\Sigma}}_{13}^3 = \underline{\mathbf{\Sigma}}_{31}^3 = \underline{\mathbf{\Sigma}}_{23}^5 = \underline{\mathbf{\Sigma}}_{32}^5 = \frac{1}{\sqrt{2}},$$

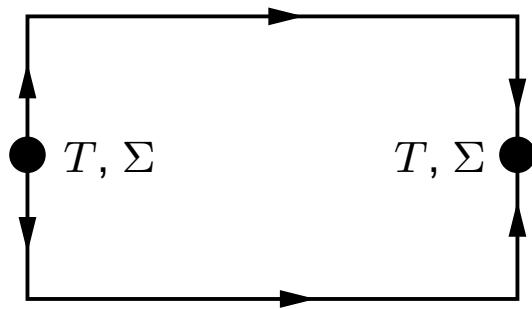
all other entries are zero.

pNRQCD for QQq

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=1}^3 \bar{q}_f i \not{D} q_f \\ & + \delta \mathcal{L}_{\text{pNRQCD}}^{(0,0)} + \delta \mathcal{L}_{\text{pNRQCD}}^{(0,1)} + \delta \mathcal{L}_{\text{pNRQCD}}^{(1,0)} + \cdots\end{aligned}$$

pNRQCD for QQq

$$\delta\mathcal{L}_{\text{pNRQCD}}^{(0,0)} = \int d^3r T^\dagger \left[iD_0 - V_T^{(0)} \right] T + \Sigma^\dagger \left[iD_0 - V_\Sigma^{(0)} \right] \Sigma$$



$$V_T^{(0)}(r) = -\frac{2}{3} \frac{\alpha_s}{|\mathbf{r}|}$$

$$V_\Sigma^{(0)}(r) = \frac{1}{3} \frac{\alpha_s}{|\mathbf{r}|}$$

pNRQCD for QQq

$$\delta\mathcal{L}_{\text{pNRQCD}}^{(0,1)} = - \int d^3r \, V_{T\boldsymbol{r}\cdot\boldsymbol{E}\Sigma}^{(0,1)} \left[\left(\sum_{ijk=1}^3 \underline{\mathbf{T}}_{ij}^\ell T_{jk}^a \underline{\boldsymbol{\Sigma}}_{ki}^\sigma \right) T^{\ell\dagger} \boldsymbol{r} \cdot g\boldsymbol{E}^a \Sigma^\sigma \right. \\ \left. - \left(\sum_{ijk=1}^3 \underline{\boldsymbol{\Sigma}}_{ij}^\sigma T_{jk}^a \underline{\mathbf{T}}_{ki}^\ell \right) \Sigma^{\sigma\dagger} \boldsymbol{r} \cdot g\boldsymbol{E}^a T^\ell \right]$$

pNRQCD for QQq

$$\begin{aligned} \delta\mathcal{L}_{\text{pNRQCD}}^{(1,0)} &= \int d^3r \, T^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} \right] T + \Sigma^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} \right] \Sigma \\ &+ V_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma}^{(1,0)} \left[\left(\sum_{ijk=1}^3 \underline{\mathbf{T}}_{ij}^\ell T_{jk}^a \underline{\boldsymbol{\Sigma}}_{ki}^\sigma \right) T^{\ell\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \right) \cdot g\boldsymbol{B}^a \Sigma^\sigma \right. \\ &\quad \left. - \left(\sum_{ijk=1}^3 \underline{\boldsymbol{\Sigma}}_{ij}^\sigma T_{jk}^a \underline{\mathbf{T}}_{ki}^\ell \right) \Sigma^{\sigma\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \right) \cdot g\boldsymbol{B}^a T^\ell \right] \\ &+ \frac{V_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}T}^{(1,0)}}{2} T^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \right) \cdot g\boldsymbol{B}^a T_{\bar{3}}^a T \\ &+ \frac{V_{\Sigma\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma}^{(1,0)}}{2} \Sigma^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)} \right) \cdot g\boldsymbol{B}^a T_6^a \Sigma + \dots \end{aligned}$$

Applications

- Spectra (m_b , m_c , B_c , η_b , ...).
- Doubly charmed baryons .
- Gluelump spectrum.
- Radiative transitions (M1,E1).
- Electromagnetic widths.
- Seminclusive radiative decays of $\Upsilon(1S)$.
- Top-antitop production near threshold at ILC.

Quarkonium Spectrum at ma_s^5

$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \langle \mathbf{E}(t) \mathbf{E}(0) \rangle(\mu)$$

Nonperturbative nonlocal condensates set the precision of the calculation.

Applications:

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Applications:

- *Perturbative spectrum calculation*

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Applications:

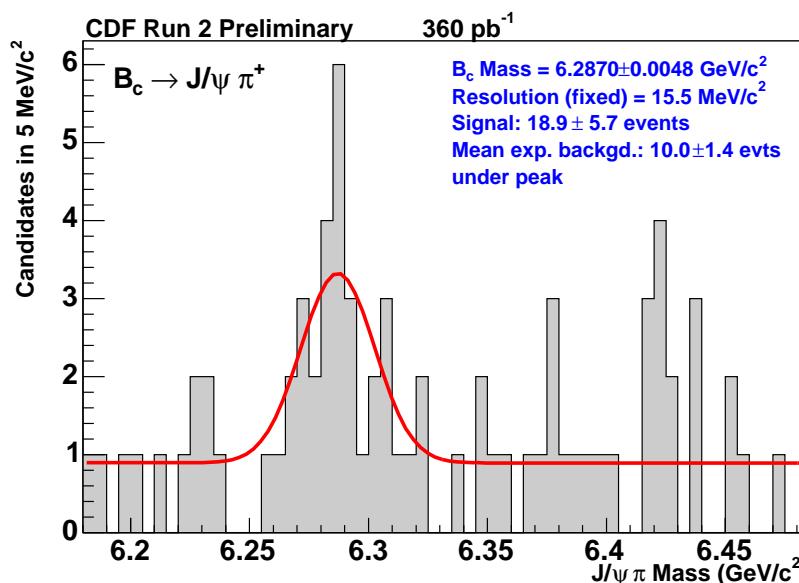
- *Perturbative spectrum calculation*
- *c and b mass extraction*

B_c mass

State	expt	lattice04	BV00	BSV01	BSV02
B_c mass (MeV)					
1^1S_0	6400(400)	6304(16)	6326(29)	6324(22)	6307(17)

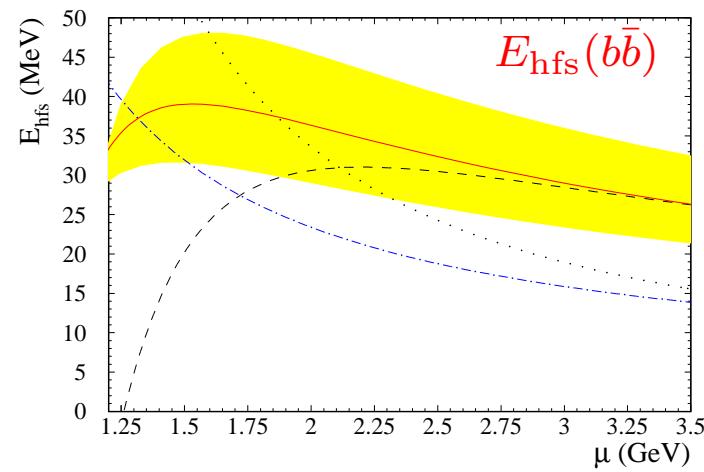
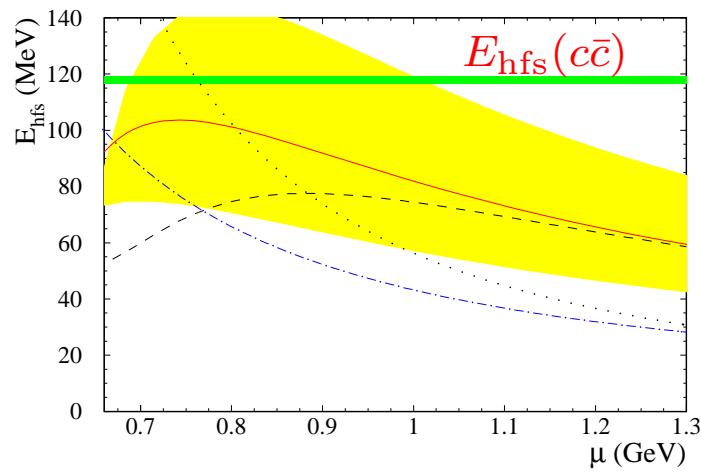
Brambilla Sumino Vairo 01 02, Brambilla Vairo 00, HPQCD-FNAL-UKQCD 04

In 2005, CDF found B_c in $B_c \rightarrow J/\psi \pi$.



$$M_{B_c} = 6287 \pm 4.8 \pm 1.1 \text{ MeV}$$

hfs and η_b mass



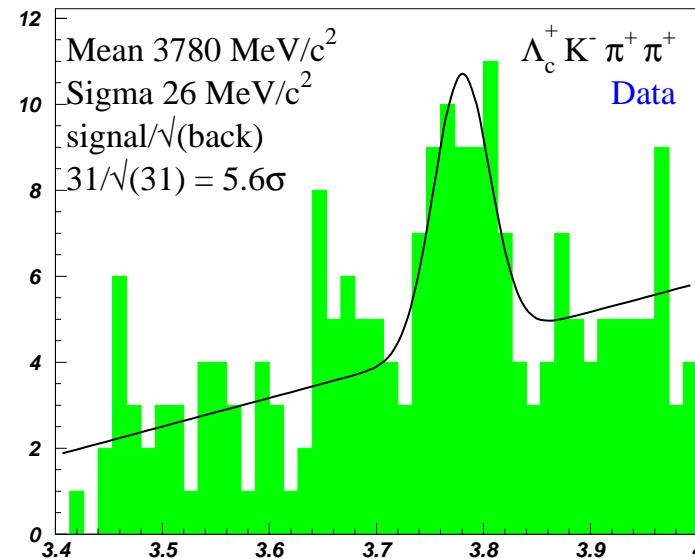
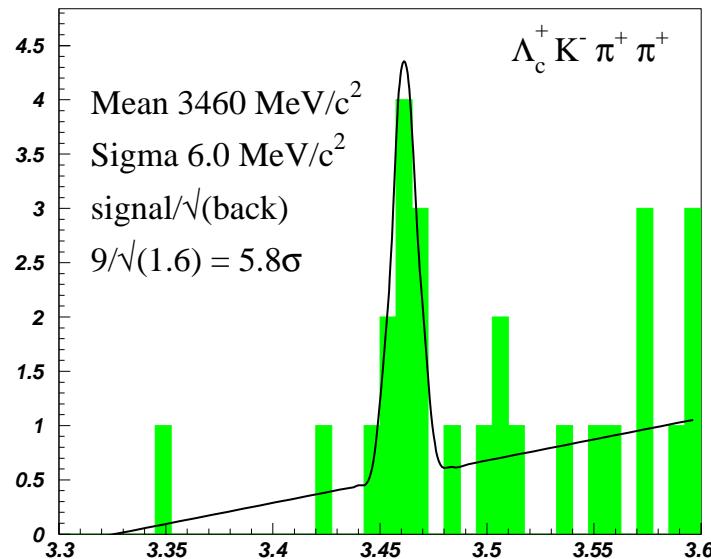
LO ...
 NLO - - -
 LL - . - . -
 NLL ———

$$M(\eta_b) = 9421 \pm 10 \text{ (th)} {}^{+9}_{-8} (\delta\alpha_s) \text{ MeV}$$

Doubly charmed baryons

Ξ_{cc} Doubly Charmed Baryons

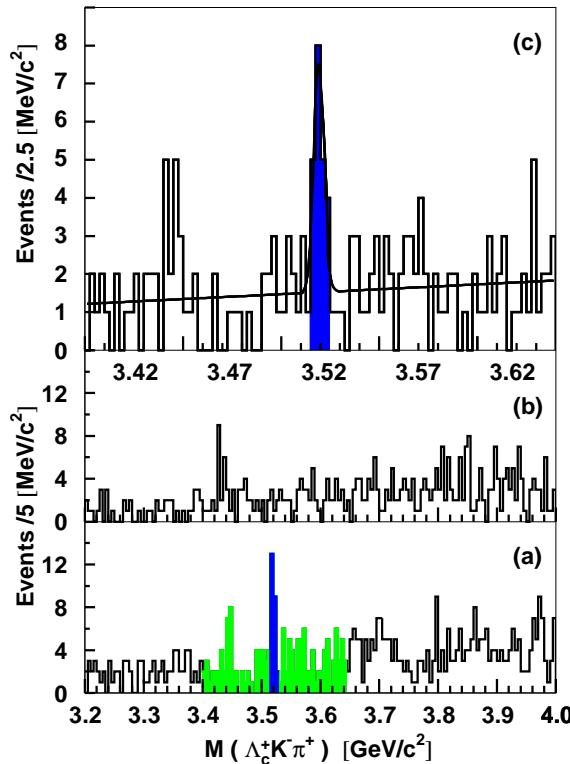
Evidence of 5 doubly charmed baryons ($ccd^+(3443)$,
 $ccd^+(3520)$, $ccu^{++}(3460)$, $ccu^{++}(3541)$, $ccu^{++}(3780)$) so far.



SELEX 02 04

Confirmation of $\Xi_{cc}^+(ccd)$ in two decay modes at a mass of $3518.7 \pm 1.7\text{MeV}$, $\tau < 33\text{fs}$

- $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$

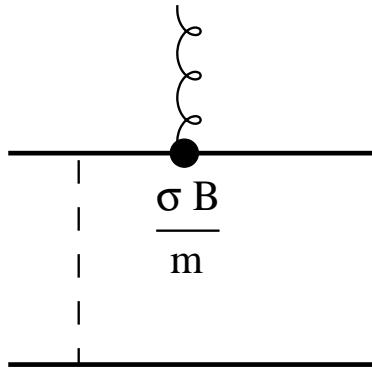


22 events in signal region, expected background 6.1

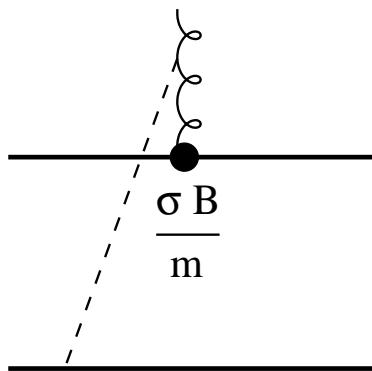
- $\Xi_{cc}^+ \rightarrow p D^+ K^-$

5.4 events in signal region, expected background 1.6

Hyperfine Splittings: NLO matching



cancel in the matching



\propto external energies $\propto p^2/m \Rightarrow$ contribute to h.o. operators

$$V_{T\boldsymbol{\sigma} \cdot \boldsymbol{B} T}^{(1,0)} = 1 + \mathcal{O}(\alpha_s^2)$$

Hyperfine Splittings: the interaction

$$\frac{c_F(\mu)}{2m} \left(1 + \mathcal{O}(\alpha_s^2)\right) T^\dagger \frac{\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}}{2} g \mathbf{B}^a T_{\bar{3}}^a T$$

Hyperfine Splittings Formula

Consider heavy-light mesons. We call P_Q and P_Q^* the $\bar{Q}u$ or $\bar{Q}d$ spin 0 or spin 1 states.

Consider the S -wave ground state of a doubly heavy baryon. Since an (anti)triplet state is antisymmetric in colour, due to the Fermi statistics, the two heavy quarks are allowed only in a spin 1 (symmetric) state. The lowest energy states for QQu or QQd are called Ξ_{QQ} (Ξ_{QQ}^*) for spin 1/2 (3/2).

$$M_{\Xi_{QQ}^*} - M_{\Xi_{QQ}} = \frac{3m_{Q'}}{4m_Q} \frac{c_F^{(Q)}}{c_F^{(Q')}} \left(M_{P_{Q'}^*} - M_{P_{Q'}} \right) \left[1 + \mathcal{O} \left(\alpha_s^2, \frac{\Lambda_{\text{QCD}}}{m_Q}, \frac{\Lambda_{\text{QCD}}}{m_{Q'}} \right) \right]$$

Savage and Wise 90; Brambilla, Roesch, Vairo 05; Fleming and Mehen 05

Hyperfine Splittings Formula

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 120 \pm 40 \text{ MeV}$$

$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 34 \pm 4 \text{ MeV}$$

Hyperfine Splittings Formula

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 120 \pm 40 \text{ MeV}$$

$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 34 \pm 4 \text{ MeV}$$

To be compared with the lattice results:

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 89 \pm 15 \text{ MeV}$$

Flynn Mescia Tariq 03 - quenched QCD

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 80 \pm 10^{+3}_{-7} \text{ MeV}$$

Lewis Mathur Woloshyn 01 - quenched QCD

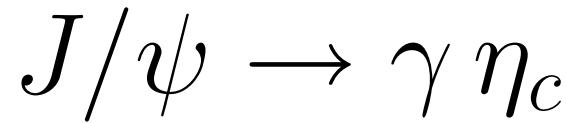
$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 20 \pm 6^{+2}_{-3} \text{ MeV}$$

Ali Khan et al. 99 - quenched NRQCD

$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 20 \pm 6^{+3}_{-4} \text{ MeV}$$

Mathur Lewis Woloshyn 02 - quenched NRQCD

Radiative Transitions (pNRQCD + ultrasoft photons)



Only one direct experimental measure:

$$\Gamma(J/\psi \rightarrow \eta_c \gamma) = (1.14 \pm 0.23) \text{ keV} \quad \text{Crystal Ball 86}$$

Moreover, there are several measurements of the BR $J/\psi \rightarrow \eta_c \gamma \rightarrow \phi \phi \gamma$ and one independent measurement of $\eta_c \rightarrow \phi \phi$ (Belle 03). From them one obtains

$$\Gamma(J/\psi \rightarrow \eta_c \gamma) = (2.9 \pm 1.5) \text{ keV}$$

Combining both

$$\Gamma(J/\psi \rightarrow \eta_c \gamma) = (1.18 \pm 0.36) \text{ keV} \quad \text{PDG 04}$$

- $\Gamma(J/\psi \rightarrow \eta_c \gamma)$ enters into many charmonium BR.
Its 30% uncertainty sets typically their experimental errors.

$$J/\psi \rightarrow \gamma \eta_c$$

At leading order $\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2.83 \text{ KeV}$

this implies:

- large value of the charm mass
- large anomalous magnetic moment of the quark
- large relativistic corrections to the S -state wave functions

M1 operator at $\mathcal{O}(1)$

$$-c_{\boldsymbol{\sigma} \cdot \mathbf{B}} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}}{2m} \right\} S$$

- $\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.
- to all orders $c_{\boldsymbol{\sigma} \cdot \mathbf{B}}$ does not get soft contributions.
- hard contributions are known:

$$c_{\boldsymbol{\sigma} \cdot \mathbf{B}} \equiv 1 + \kappa_c = 1 + \frac{2\alpha_s(m_c)}{3\pi} + \dots$$

- No large quarkonium anomalous magnetic moment!
(see also the lattice calculation of Dudek Edwards Richards 06)

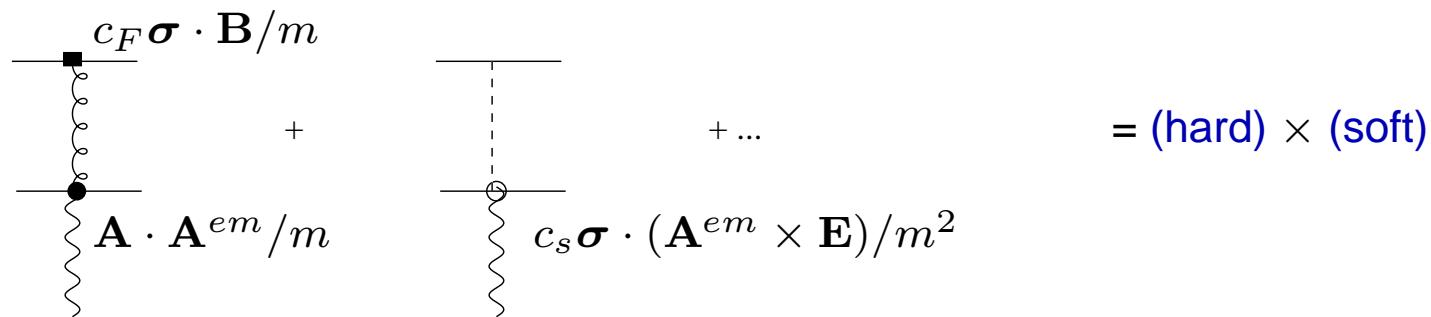
M1 operators at $\mathcal{O}(v^2)$

$$-c_{p^2}\boldsymbol{\sigma}\cdot\mathbf{B}\left\{S^{\dagger},\frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{4m^3}\right\}\boldsymbol{\nabla}_r^2S$$

- $c_{p^2}\boldsymbol{\sigma}\cdot\mathbf{B}=1$

M1 operators at $\mathcal{O}(v^2)$

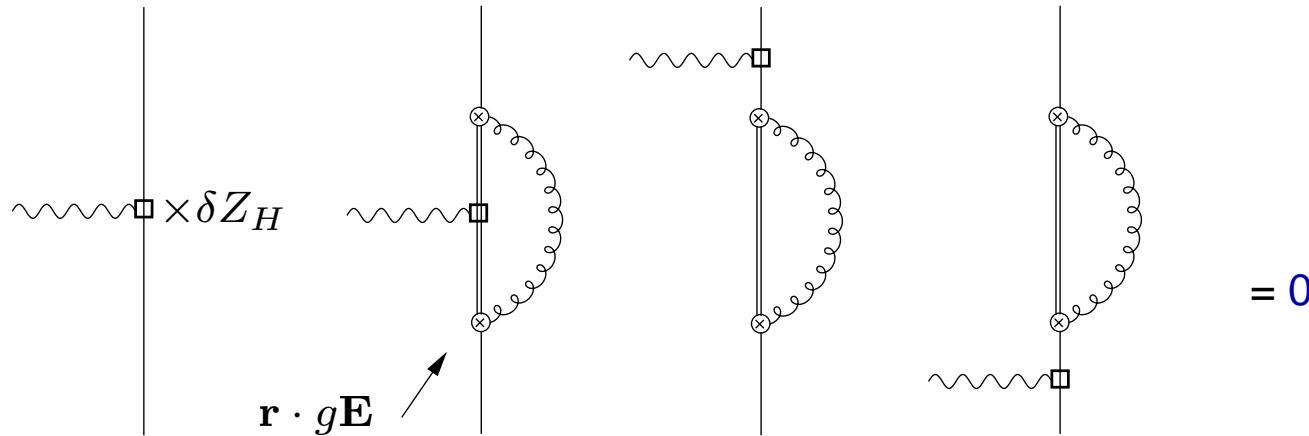
$$-c_r \boldsymbol{\sigma} \cdot \mathbf{B} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e \mathbf{B}^{em}}{12m^2} \right\} S$$



- to all orders (hard) = $2c_F - c_s = 1$; (soft) = $-rV'_s$
(due to reparametrization/Poincaré invariance) Brambilla Gromes Vairo 03
- Therefore $c_r \boldsymbol{\sigma} \cdot \mathbf{B} = -rV'_s$
- No scalar interaction!

M1 operators at $\mathcal{O}(v^2)$

Coupling of photons with octets: $-c_{\sigma \cdot \mathbf{B}} \left\{ O^\dagger, \frac{\sigma \cdot e\mathbf{B}^{em}}{2m} \right\} O$



- If $mv^2 \sim \Lambda_{QCD}$ the above graphs are potentially of order $\Lambda_{QCD}^2/(mv)^2 \sim v^2$.
- The contribution vanishes because $\sigma \cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.
- There are no non-perturbative contributions at $\mathcal{O}(v^2)$!

$$J/\psi \rightarrow \gamma \eta_c$$

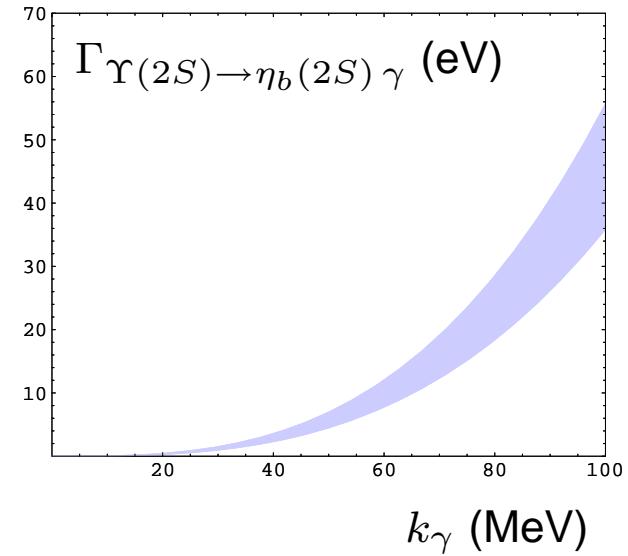
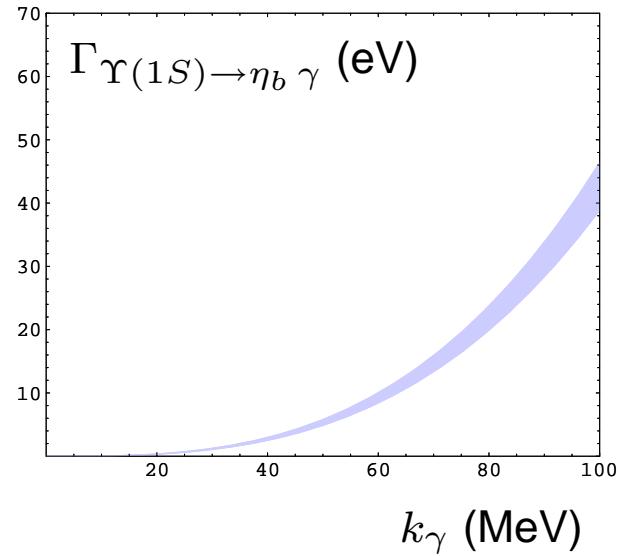
Up to order v^2 the transition $J/\psi \rightarrow \gamma \eta_c$ is completely accessible by perturbation theory.

$$\Gamma(J/\psi \rightarrow \gamma \eta_c) = \frac{16}{3} \alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2} \left[1 + C_F \frac{\alpha_s(M_{J/\psi}/2)}{\pi} - \frac{2}{3} (C_F \alpha_s(p_{J/\psi}))^2 \right]$$

The normalization scale for the α_s inherited from κ_c is the charm mass ($\alpha_s(M_{J/\psi}/2) \approx 0.35 \sim v^2$), and for the α_s , which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx m C_F \alpha_s(p_{J/\psi})/2 \approx 0.8 \text{ GeV} \sim m v$.

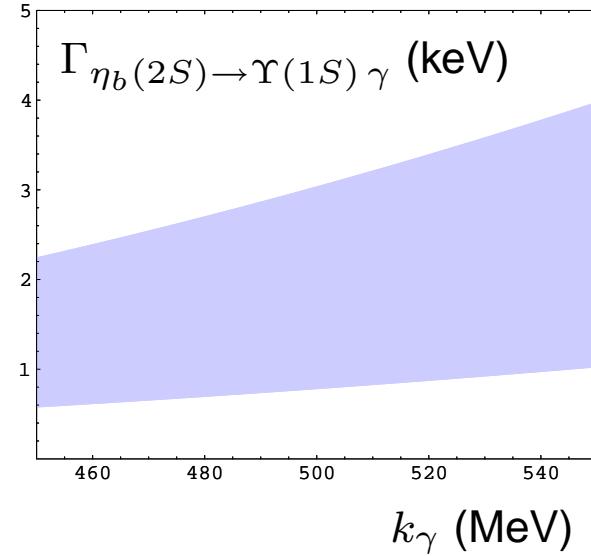
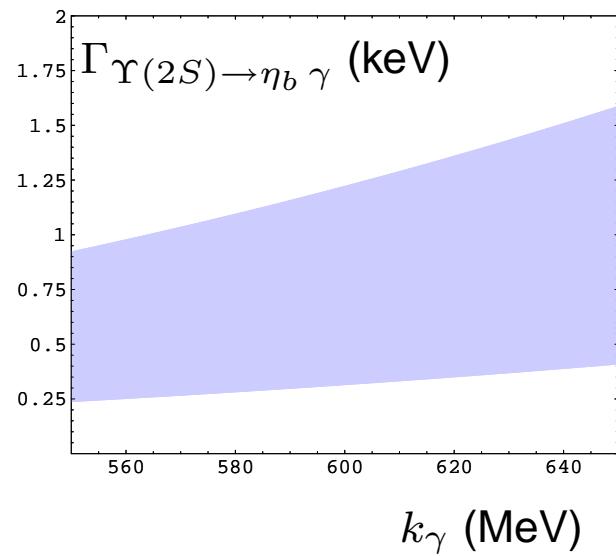
$$\Gamma(J/\psi \rightarrow \eta_c \gamma) = (1.5 \pm 1.0) \text{ keV.}$$

$\Gamma_{\Upsilon(1S) \rightarrow \eta_b \gamma}$ and $\Gamma_{\Upsilon(2S) \rightarrow \eta_b(2S) \gamma}$

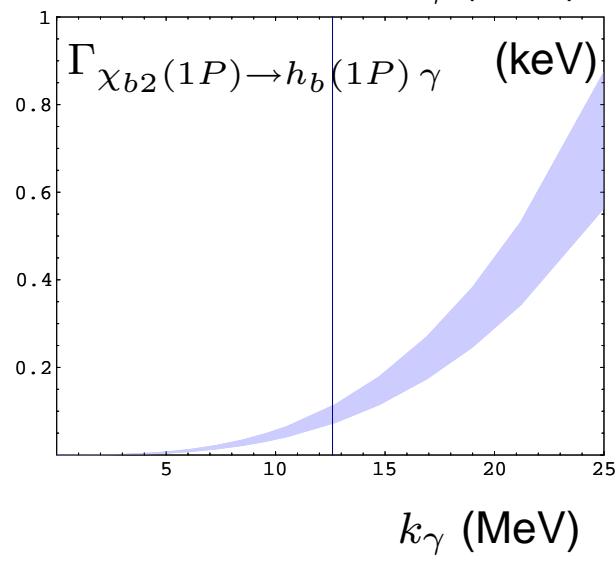
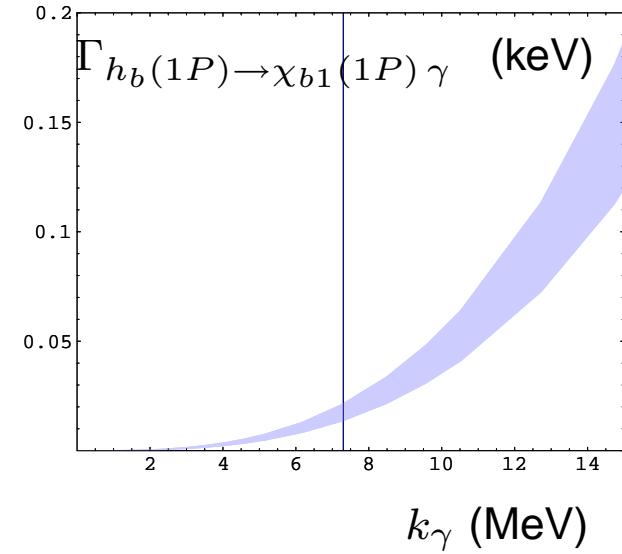
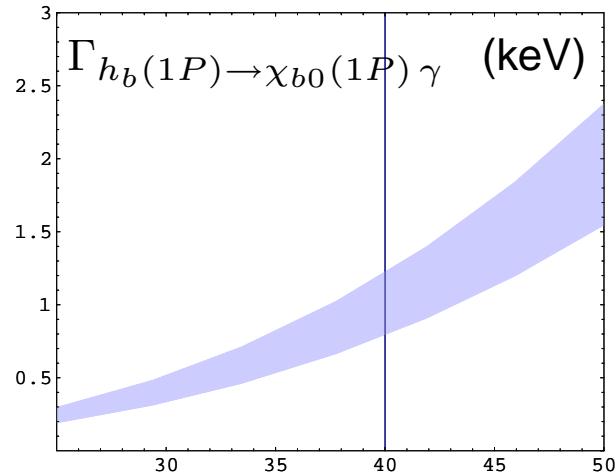


$$\mathcal{B}_{\Upsilon(1S) \rightarrow \eta_b \gamma} = (6.8 \pm 5.5) \times 10^{-5}$$

$\Gamma_{\Upsilon(2S) \rightarrow \eta_b \gamma}$ and $\Gamma_{\eta_b(2S) \rightarrow \Upsilon(1S) \gamma}$



$\Gamma_{h_b(1P) \rightarrow \chi_{b0}(1P)\gamma}$, $\Gamma_{h_b(1P) \rightarrow \chi_{b1}(1P)\gamma}$
 and $\Gamma_{\chi_{b2}(1P) \rightarrow h_b(1P)\gamma}$



$$\Gamma_{h_b(1P) \rightarrow \chi_{b0}(1P)\gamma} = 1 \pm 0.2 \text{ keV}$$

$$\Gamma_{h_b(1P) \rightarrow \chi_{b1}(1P)\gamma} = 17 \pm 4 \text{ eV}$$

$$\Gamma_{\chi_{b2}(1P) \rightarrow h_b(1P)\gamma} = 90 \pm 20 \text{ eV}$$

Strongly coupled pNRQCD

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

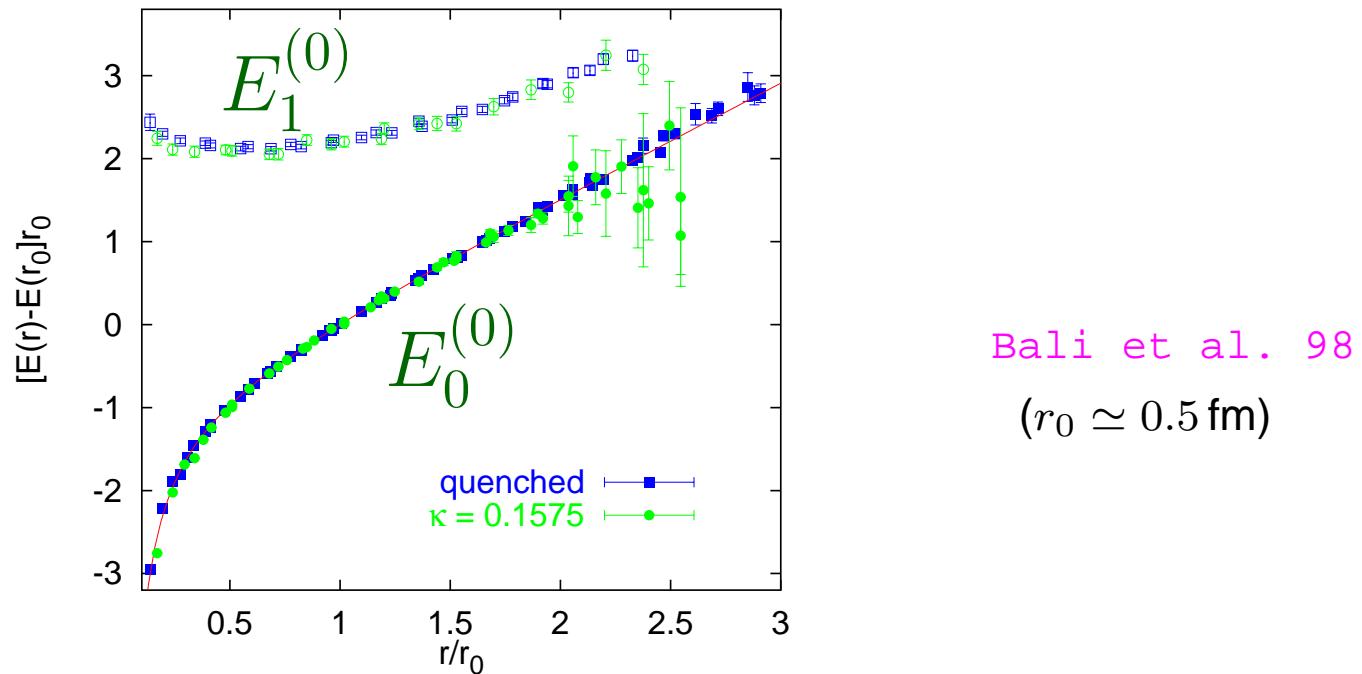
- All **scales above** mv^2 are integrated out.

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

- All scales above mv^2 are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order Λ_{QCD} with the static $Q\bar{Q}$ energy.

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

- All scales above mv^2 are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order Λ_{QCD} with the static $Q\bar{Q}$ energy.



pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

- All scales above mv^2 are integrated out.
 - All gluonic excitations between heavy quarks are integrated out since they develop a gap of order Λ_{QCD} with the static $Q\bar{Q}$ energy.
- ⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

$$\mathcal{L} = \text{Tr} \left\{ \textcolor{blue}{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \textcolor{blue}{S} \right\}$$

Brambilla Pineda Soto Vairo 00

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

$$\mathcal{L} = \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\}$$

Brambilla Pineda Soto Vairo 00

- The potential V_s ($\text{Re } V_s + i \text{Im } V_s$) is non-perturbative:
 - (a) to be determined from the lattice;
 - (b) to be determined from QCD vacuum models.

Creutz et al. 82, Campostrini 85, Michael 85, Born et al. 94,
Bali Schilling Wachter 97, Brambilla et al. 93, 95, 97, 98

pNRQCD for $mv \sim \Lambda_{\text{QCD}}$

$$\mathcal{L} = \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S \right\}$$

Brambilla Pineda Soto Vairo 00

- The matching condition is:

$$\langle H | \mathcal{H} | H \rangle = \langle nljs | \frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n} | nljs \rangle$$

Applications

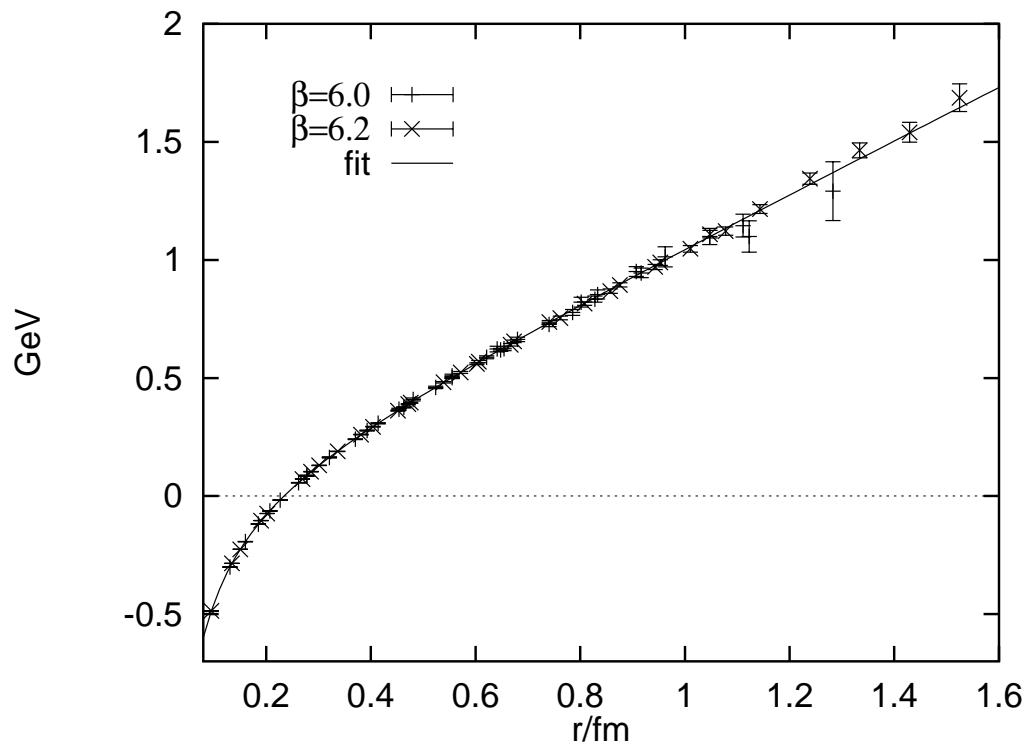
- Nonperturbative potentials and spectrum.
- Inclusive decays .

The non-perturbative Potentials

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m} + \frac{V_s^{(2)}}{m^2} + \dots$$

The non-perturbative Potentials

$$V_s^{(0)} = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle W(r \times T) \rangle = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle \boxed{\square} \rangle$$



Bali Schilling Wachter 97

The non-perturbative Potentials

$$V_s^{(1)} = -{}^{(0)}\langle 0 | \mathbf{D}^2 | 0 \rangle^{(0)} = -\nabla^2 + \sum_{k \neq 0} \left| \frac{{}^{(0)}\langle k | g \mathbf{E} | 0 \rangle^{(0)}}{E_0^{(0)} - E_k^{(0)}} \right|^2$$

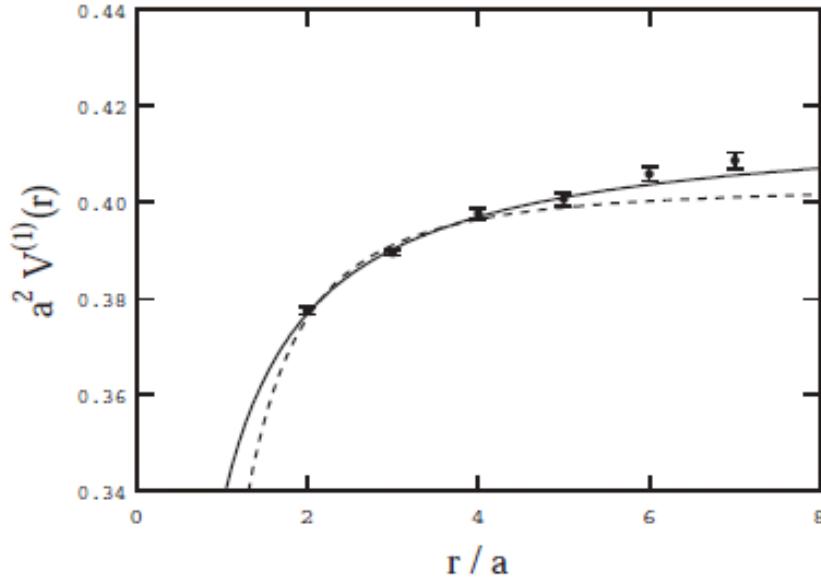
kinetic energy \uparrow

Since

$$\langle\langle \mathbf{E}(\mathbf{t}) \cdot \mathbf{E}(0) \rangle\rangle_{\square} = \sum_k |{}^{(0)}\langle 0 | g \mathbf{E} | k \rangle^{(0)}|^2 e^{-i E_0^{(0)} T - i(E_k^{(0)} - E_0^{(0)})\mathbf{t}}$$

$$V_s^{(1)} = -\frac{1}{2} \int_0^\infty dt t \langle \boxed{\mathbf{E}} \rangle$$

The non-perturbative Potentials



$$V_s^{(1)} = -\frac{C}{r} + a$$

Correction comparable to the Coulombic term of the static potential for charmonium and 26% of it for bottomonium!

Koma, Koma and Wittig 2006 (Preliminary)

The non-perturbative Potentials

$$V_{\text{SD}}^{(2)} = \frac{1}{r} \left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt t \langle \begin{array}{c} \text{E} \\ \text{i} \quad \text{j} \\ \text{B} \end{array} \rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \begin{array}{c} \text{B} \\ \text{i} \quad \text{j} \\ \text{B} \end{array} \rangle - \frac{\delta_{ij}}{3} \langle \begin{array}{c} \text{B} \\ \text{B} \end{array} \rangle \right)$$

$$\times \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right)$$

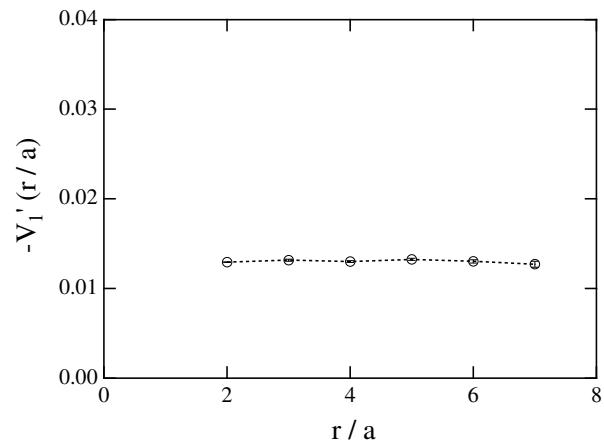
$$+ \left(\frac{2}{3} c_F^2 i \int_0^\infty dt \langle \begin{array}{c} \text{B} \\ \text{B} \end{array} \rangle - 4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2$$

The non-perturbative Potentials

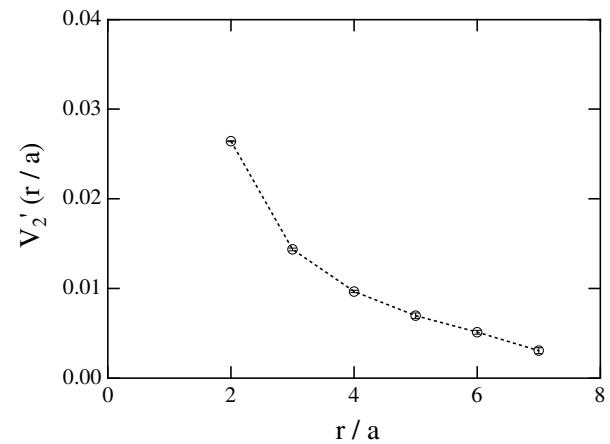
$$\begin{aligned}
V_{\text{SI}}^{(2)} = & p^i \left(i \int_0^\infty dt t^2 \langle \boxed{\overset{\bullet}{i} \quad \overset{\bullet}{j}} \rangle + \langle \boxed{i \quad \overset{\bullet}{j}} \rangle \right) p^j \\
& - \frac{c_F^2}{2} i \int_0^\infty dt \langle \boxed{\overset{\bullet}{i} \quad \overset{\bullet}{j}} \rangle + (d_1 + C_F d_3 + \pi C_F \alpha_s c_D) \delta^{(3)}(\mathbf{r}) \\
& - i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\langle \boxed{\overset{\bullet}{i} \quad \overset{\bullet}{j} \quad \overset{\bullet}{k}} \rangle + \langle \boxed{\overset{\bullet}{i} \quad \overset{\bullet}{j} \quad \overset{\bullet}{k}} \rangle \right) \\
& + \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i \\
& \times \left(\langle \boxed{\overset{\bullet}{i} \quad \overset{\bullet}{j} \quad \overset{\bullet}{k}} \rangle + \frac{1}{2} \langle \boxed{i \quad \overset{\bullet}{j} \quad \overset{\bullet}{k}} \rangle + \frac{1}{2} \langle \boxed{i \quad \overset{\bullet}{j} \quad \overset{\bullet}{k}} \rangle \right) \\
& - 2 b_3 f_{abc} \int d^3 \mathbf{x} g \langle\langle G_{\mu\nu}^a(\mathbf{x}) G_{\mu\alpha}^b(\mathbf{x}) G_{\nu\alpha}^c(\mathbf{x}) \rangle\rangle_{\square}^c
\end{aligned}$$

Spin-dependent potentials from the lattice

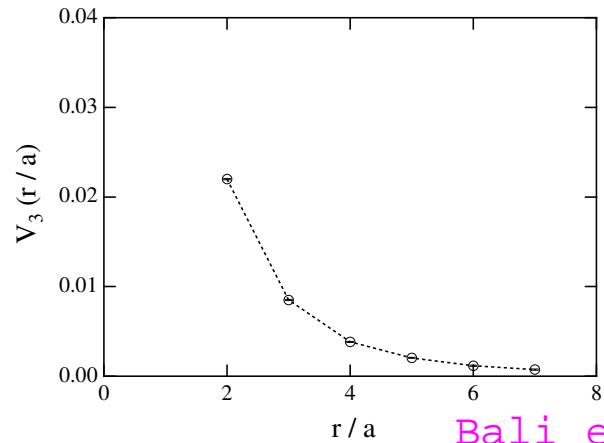
$$2 \int_0^\infty d\tau \tau \langle\langle B_y(\mathbf{r}, 0) E_z(\mathbf{r}, \tau) \rangle\rangle$$



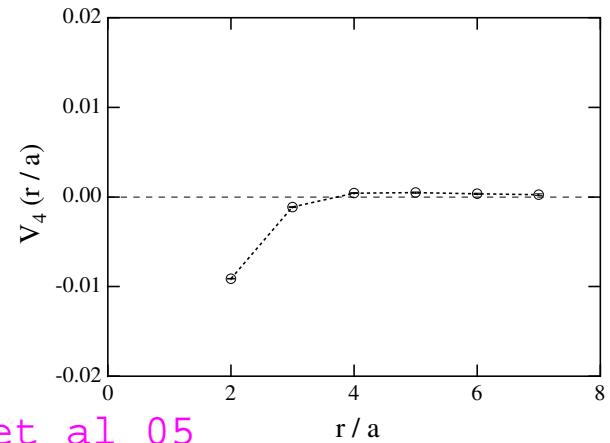
$$2 \int_0^\infty d\tau \tau \langle\langle B_y(\mathbf{0}, 0) E_z(\mathbf{r}, \tau) \rangle\rangle$$



$$2 \int_0^\infty d\tau [\langle\langle B_x(\mathbf{0}, 0) B_x(\mathbf{r}, \tau) \rangle\rangle - (x \rightarrow y)]$$



$$2 \int_0^\infty d\tau [\langle\langle B_x(\mathbf{0}, 0) B_x(\mathbf{r}, \tau) \rangle\rangle + 2(x \rightarrow y)]$$



Bali et al 97, Koma et al 05

- The power counting dictates the quantum-mechanical perturbation theory.

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- Quantum mechanical divergences (like the one coming from iterations of delta potentials) are absorbed by NRQCD matching coefficients.

- The power counting dictates the quantum-mechanical perturbation theory.
- Quantum mechanical divergences (like the one coming from iterations of delta potentials) are absorbed by NRQCD matching coefficients.
- You do not need to repeat a lattice evaluation for each quarkonium state.

Conclusions

- NREFTs enable a systematic and under control study of heavy systems inside QCD, in particular give a field theoretical definition of the potentials.

Heavy quarkonium becomes

- a competitive source for some of the SM parameters:
 $m_t, m_b, m_c, \alpha_s, \dots$
- a privileged system to study the interplay of perturbative and non-perturbative QCD.

Future Perspectives

Spectra

Future Perspectives

Spectra

- *pNRQCD on the lattice*
- *EFT for states close to threshold (hybrids, X, Y, Z ...)*

Future Perspectives

Spectra

Decays

Future Perspectives

Spectra

Decays

- *Theory of Radiative Transitions: E transitions, multipolarity*
- *Hadronic decays: 15% rule and its violations*

Future Perspectives

Spectra

Decays

Production

Future Perspectives

Spectra

Decays

Production

- *Polarization at high p_t*
- *Double charmonium production*

Future Perspectives

Spectra

Decays

Production

in Media

Future Perspectives

Spectra

Decays

Production

in Media

- QCD and EFT at finite T : RG equations (T, gT, g^2T, \dots)
- Charmonium suppression patterns

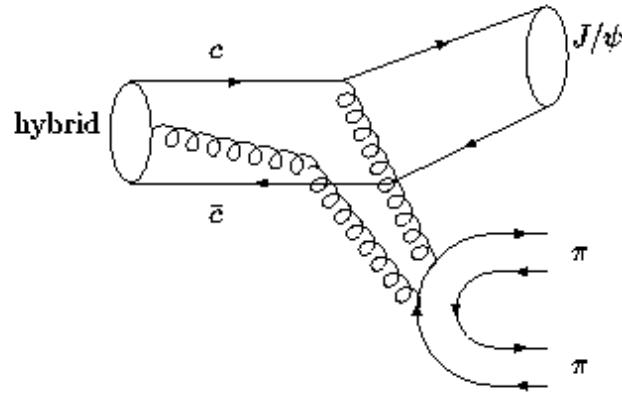
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N. Brambilla, M. Krämer, R. Musso, A. Vairo *et al.*
Heavy Quarkonium Physics
CERN Yellow Report; [arXiv:hep-ph/0412158](https://arxiv.org/abs/hep-ph/0412158).
- N. Brambilla, A. Pineda, J. Soto and A. Vairo
Effective field theories for heavy quarkonium
Rev. Mod. Phys. 77 (2005) 1423; [arXiv:hep-ph/0410047](https://arxiv.org/abs/hep-ph/0410047).

Hybrids

As an example, we consider the $\text{Y}(4260)$ case and a possible hybrid interpretation of it.

Kou Pene 05

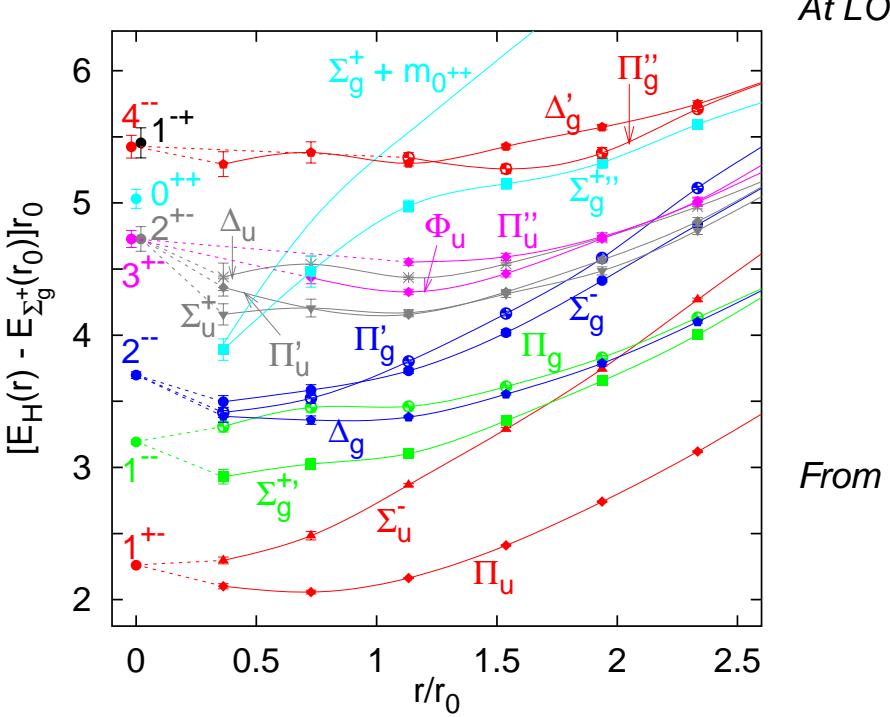


$\text{Y}(4260)$ is generated from initial state radiation in $e^+e^- \rightarrow \gamma J/\psi \pi^+\pi^- \Rightarrow J^{PC} = 1^{--}$

$$|Y\rangle = |\Pi_u\rangle \otimes |\phi\rangle$$

- $|\Pi_u\rangle$ is a 1^{+-} static hybrid state.
- $|\phi\rangle$ ($P = -1$) is the solution of the Schrödinger equation whose potential is the static energy of $|\Pi_u\rangle$.

Hybrids



Juge Kuti Morningstar 00, 03

At LO in the multipole expansion

$$H = e^{-iT} E_H$$

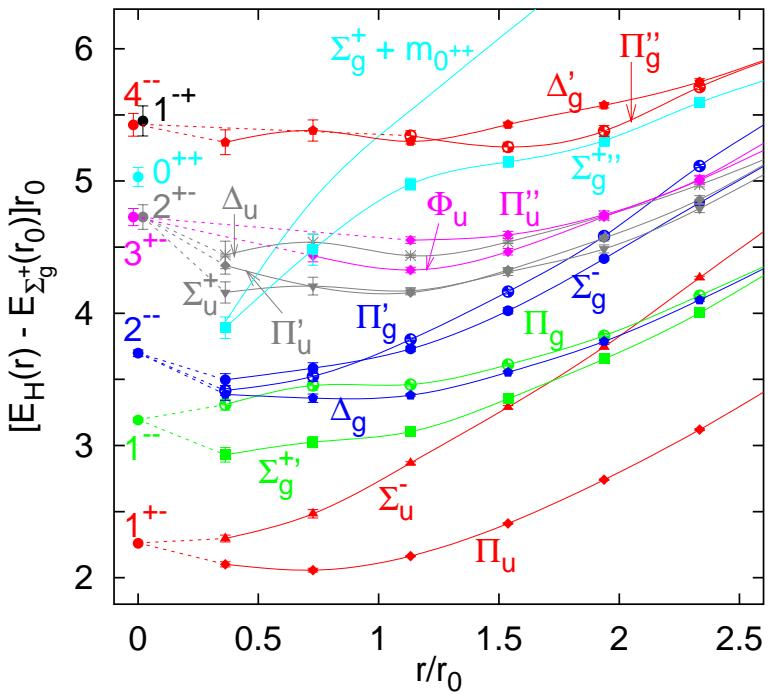
$$E_H = V_o + \frac{i}{T} \ln \langle H^a \left(\frac{T}{2} \right) \phi_{ab}^{\text{adj}} H^b \left(-\frac{T}{2} \right) \rangle$$

From

$$\langle H^a \left(\frac{T}{2} \right) \phi_{ab}^{\text{adj}} H^b \left(-\frac{T}{2} \right) \rangle^{\text{np}} \sim h e^{-iT\Lambda_H}$$

$$E_H(\mathbf{r}) = V_o(\mathbf{r}) + \Lambda_H$$

Hybrids



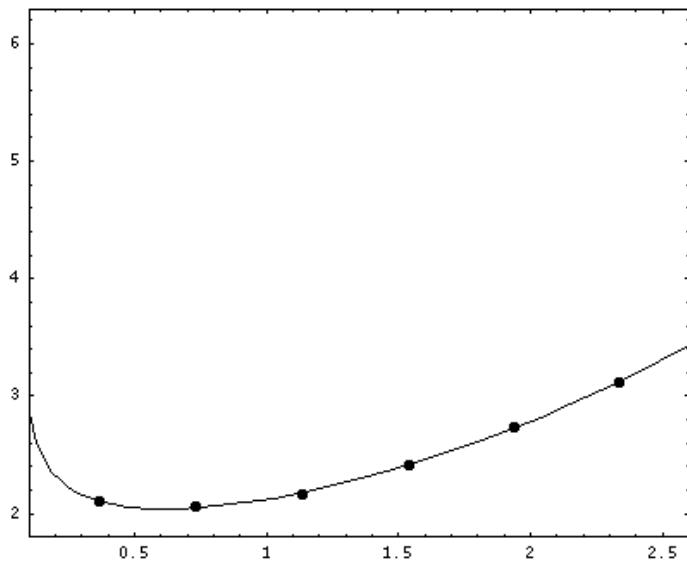
	$L = 1$	$L = 2$
$\Sigma_g^{+'}$	$\mathbf{r} \cdot (\mathbf{D} \times \mathbf{B})$	
Σ_g^-		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{B})$
Π_g	$\mathbf{r} \times (\mathbf{D} \times \mathbf{B})$	
Π'_g		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\mathbf{r} \cdot \mathbf{B}))$
Δ_g		$(\mathbf{r} \times \mathbf{D})^i(\mathbf{r} \times \mathbf{B})^j +$ $+ (\mathbf{r} \times \mathbf{D})^j(\mathbf{r} \times \mathbf{B})^i$
Σ_u^+		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{E})$
Σ_u^-	$\mathbf{r} \cdot \mathbf{B}$	
Π_u	$\mathbf{r} \times \mathbf{B}$	
Π'_u		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\mathbf{r} \cdot \mathbf{E}))$
Δ_u		$(\mathbf{r} \times \mathbf{D})^i(\mathbf{r} \times \mathbf{E})^j +$ $+ (\mathbf{r} \times \mathbf{D})^j(\mathbf{r} \times \mathbf{E})^i$

Hybrids

J^{PC}	H	$\Lambda_H^{\text{RS}} r_0$	$\Lambda_H^{\text{RS}}/\text{GeV}$
1^{+-}	B_i	2.25(39)	0.87(15)
1^{--}	E_i	3.18(41)	1.25(16)
2^{--}	$D_{\{i} B_{j\}}$	3.69(42)	1.45(17)
2^{+-}	$D_{\{i} E_{j\}}$	4.72(48)	1.86(19)
3^{+-}	$D_{\{i} D_j B_{k\}}$	4.72(45)	1.86(18)
0^{++}	\mathbf{B}^2	5.02(46)	1.98(18)
4^{--}	$D_{\{i} D_j D_k B_{l\}}$	5.41(46)	2.13(18)
1^{-+}	$(\mathbf{B} \wedge \mathbf{E})_i$	5.45(51)	2.15(20)

Foster Michael 99, Brambilla Pineda Soto Vairo 99, Bali Pineda 03

Hybrids



Fitting the Π_u curve, $E_{\Pi_u} = (0.87 + 0.11/r + 0.24 r^2)$ GeV
and solving the Schrödinger equation, one gets

$$M(Y) = 2 \times 1.48 + 0.87 + 0.53 = 4.36 \text{ GeV}$$

Exclusive Decay Modes

$$\kappa[h_1 h_2] = \frac{\mathcal{B}(\psi(2S) \rightarrow h_1 h_2)}{\mathcal{B}(J/\psi \rightarrow h_1 h_2)} \frac{\mathcal{B}(J/\psi \rightarrow e^+ e^-)}{\mathcal{B}(\psi(2S) \rightarrow e^+ e^-)} \frac{\varrho[J/\psi h_1 h_2]}{\varrho[\psi(2S) h_1 h_2]},$$

- The phase-space factor $\rho \approx 1$.
- If $\Gamma(J/\psi \rightarrow h_1 h_2) \approx |\psi_{J/\psi}(\mathbf{r} = 0)|^2 |\mathcal{A}(c(\mathbf{0})\bar{c}(\mathbf{0}) \rightarrow h_1 h_2)|^2 \frac{\varrho[J/\psi h_1 h_2]}{16\pi M_{J/\psi}}$ and analogously for the $\psi(2S)$.

Then

$$\kappa[h_1 h_2] \approx 1$$

Also known as 15% rule.

Exclusive Decay Modes

Decay mode $h_1 h_2$ PDG 03	$\mathcal{B}[J/\psi \rightarrow h_1 h_2]$ ($\times 10^4$)	$\mathcal{B}[\psi' \rightarrow h_1 h_2]$ ($\times 10^4$)	$\kappa[h_1 h_2]$
$\varrho\pi$	127 ± 9	$< 0.83 (< 0.28)$	$< 0.054 (< 0.18)$
$\omega\pi^0$	4.2 ± 0.6	$0.38 \pm 0.17 \pm 0.11$	0.7 ± 0.4
$\varrho\eta$	1.93 ± 0.23		
$\omega\eta$	15.8 ± 1.6	< 0.33	< 0.17
$\phi\eta$	6.5 ± 0.7		
$\varrho\eta'(958)$	1.05 ± 0.18		
$\omega\eta'(958)$	1.67 ± 0.25		
$\phi\eta'(958)$	3.3 ± 0.4		
$K^*(892)^\mp K^\pm$	50 ± 4	$< 0.54 (< 0.30)$	$< 0.089 (< 0.049)$
$\bar{K}^*(892)^0 K^0 + \text{c.c.}$	42 ± 4	$0.81 \pm 0.24 \pm 0.16$	0.15 ± 0.05
$\pi^\pm b_1(1235)^\mp$	30 ± 5	3.2 ± 0.8	0.79 ± 0.24
$\pi^0 b_1(1235)^0$	23 ± 6		
$K^\pm K_1(1270)^\mp$	< 30	10.0 ± 2.8	> 1.7
$K^\pm K_1(1400)^\mp$	38 ± 14	< 3.1	< 0.78

Exclusive Decay Modes

Decay mode $h_1 h_2$ PDG 04, BES 04, CLEO 04	$\mathcal{B}(J/\psi \rightarrow h_1 h_2)$ ($\times 10^4$)	$\mathcal{B}(\psi' \rightarrow h_1 h_2)$ ($\times 10^4$)	$\kappa[h_1 h_2]$
$\varrho\pi$	127 ± 9	0.46 ± 0.09	0.028 ± 0.006
$\omega\pi^0$	4.2 ± 0.6	0.22 ± 0.09	0.40 ± 0.17
$\varrho\eta$	1.93 ± 0.23	0.23 ± 0.12	0.9 ± 0.5
$\omega\eta$	15.8 ± 1.6	< 0.11	< 0.06
$\phi\eta$	6.5 ± 0.7	0.35 ± 0.11	0.40 ± 0.13
$\varrho\eta'(958)$	1.05 ± 0.18	$0.19_{-0.11}^{+0.16} \pm 0.03$	2.5 ± 0.9
$\omega\eta'(958)$	1.67 ± 0.25	< 0.81	< 4.3
$\phi\eta'(958)$	3.3 ± 0.4	$0.33 \pm 0.13 \pm 0.07$	0.71 ± 0.33
$K^*(892)^\mp K^\pm$	50 ± 4	0.26 ± 0.11	0.039 ± 0.017
$\bar{K}^*(892)^0 K^0 + \text{c.c.}$	42 ± 4	1.55 ± 0.25	0.28 ± 0.05
$\pi^\pm b_1(1235)^\mp$	30 ± 5	3.9 ± 1.6	1.0 ± 0.4
$\pi^0 b_1(1235)^0$	23 ± 6	$4.0_{-0.8}^{+0.9} \pm 0.6$	1.3 ± 0.5
$K^\pm K_1(1270)^\mp$	< 30	10.0 ± 2.8	> 1.7
$K^\pm K_1(1400)^\mp$	38 ± 14	< 3.1	< 0.8

Exclusive Decay Modes

Possible explanations include:

- suppression of the $c\bar{c}$ wave function at the origin for a component of $\psi(2S)$ in which the $c\bar{c}$ is in a color-octet 3S_1 state.
- suppression of the $\omega\phi$ component of $\psi(2S)$.
- cancellation between $c\bar{c}$ and $D\bar{D}$ components of $\psi(2S)$.
- cancellation between $c\bar{c}$ and glueball components of $\psi(2S)$.
- cancellation between S -wave $c\bar{c}$ and D -wave $c\bar{c}$ components of $\psi(2S)$.
- cancellation between the amplitudes for the resonant process $e^+e^- \rightarrow \psi(2S) \rightarrow \rho\pi$ and the direct process $e^+e^- \rightarrow \rho\pi$.

Quarkonium Production

- There is no formal proof of the NRQCD factorization yet.
- The relevant 4-fermion operators are

$$\psi^\dagger \textcolor{red}{K}^{(n)} \chi a_H^\dagger a_H \chi^\dagger \textcolor{red}{K'}^{(n)} \psi$$

Recently it has been proved that the cancellation of the IR divergences at NNLO requires the modification of the 4 fermion operators into

$$\begin{aligned} & \psi^\dagger \textcolor{red}{K}^{(n)} \chi \phi_l^\dagger(0, \infty) a_H^\dagger a_H \phi_l(0, \infty) \chi^\dagger \textcolor{red}{K'}^{(n)} \psi \\ & \phi_l(0, \infty) = P \exp \left(-ig \int_0^\infty d\lambda l \cdot A(\lambda l) \right), \quad l^2 = 1 \end{aligned}$$

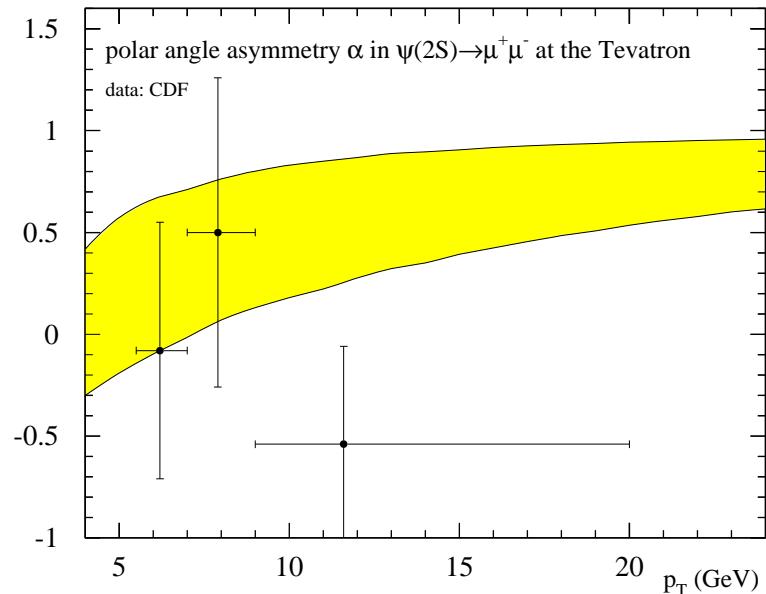
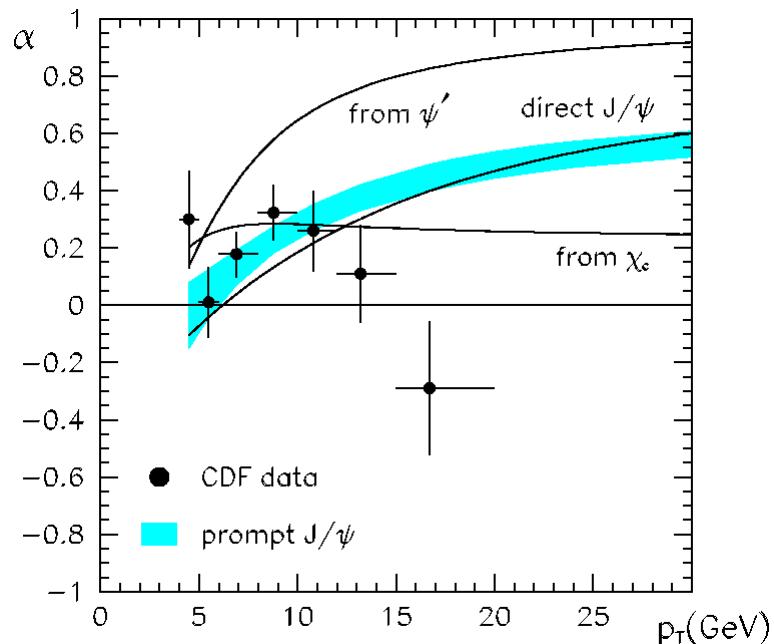
Nayak Qiu Sterman 05

This may open the way to a pNRQCD factorization for production processes.

Charmonium Polarization at the Tevatron

- For large p_T quarkonium production, gluon fragmentation via the color-octet mechanism dominates: $\langle O_8^{J/\psi}(^3S_1) \rangle$.
- At large p_T the gluon is nearly on mass shell and so is transversely polarized.
- In color octet gluon fragmentation, most of the gluon's polarization is transferred to the J/ψ .
- Radiative corrections, color singlet production dilute this.
- In the case of the J/ψ feeddown is important:
feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
- feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- *Spin-flippling terms are assumed suppressed. But this strictly depends on the power counting.
If they are not, polarization may dilute at high p_T .*

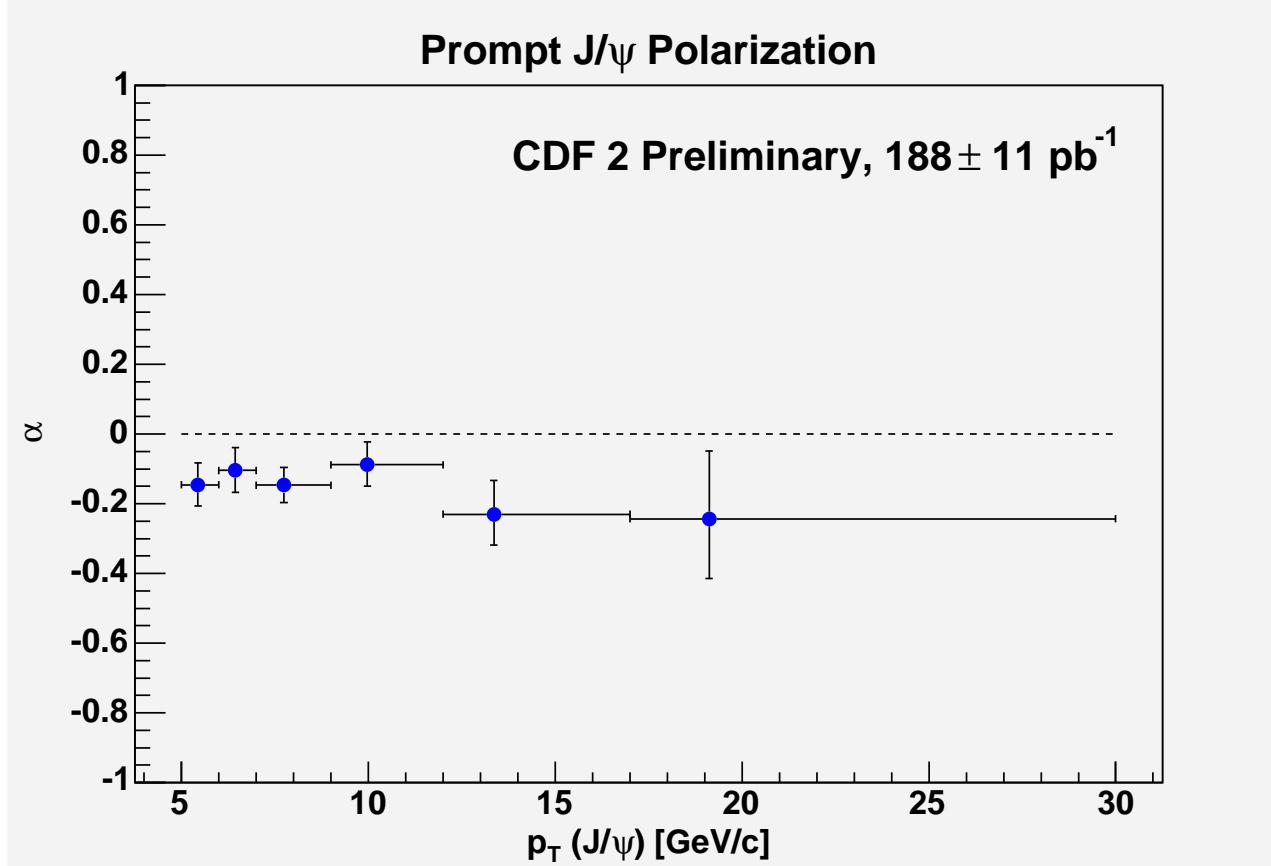
Charmonium Polarization at the Tevatron



$$\frac{d\sigma}{d \cos \theta} \propto 1 + \alpha \cos^2 \theta$$

$\alpha = 1$ is completely transverse $\alpha = -1$ is completely longitudinal.

Charmonium Polarization at the Tevatron



CDF (preliminary) 05

Double Charmonium Production

$$\sigma(e^+e^- \rightarrow J/\psi + \eta_c) \gtrsim 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad \text{Belle 04}$$

$$\sigma(e^+e^- \rightarrow J/\psi + \eta_c) \gtrsim 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad \text{BaBar 05}$$

$$\sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 3.78 \pm 1.26 \text{ fb} \quad \text{NRQCD}$$

- Includes QED interference corrections (-21%)
- Includes uncertainties from h.o. in α_s , v and matrix elements
- In Belle 04 $\sigma(e^+e^- \rightarrow J/\psi + J/\psi) < 9.1 \text{ fb}$
- In Brodsky et al 03 $\sigma(e^+e^- \rightarrow J/\psi + \mathcal{G}_J) \approx 1.4 \text{ fb}$
where \mathcal{G}_J is a J^{++} glueball, $J = 0, 2$

Double Charmonium Production

$$\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14$$

Belle 03

$$\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} \approx 0.1$$

Cho Leibovich 96, Baek et al 96, Yuan et al 96

Double Charmonium Production

$$\sigma(e^+e^- \rightarrow J/\psi + J/\psi) < 9.1 \text{ fb}$$

Belle 04

$$\sigma(e^+e^- \rightarrow J/\psi + J/\psi) = 8.70 \pm 2.94 \text{ fb}$$

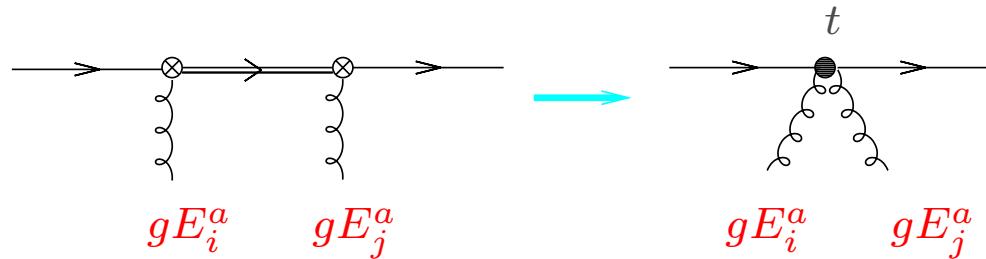
Bodwin et al 03

Heavy quarkonium in media

One may address in the framework of pNRQCD:

- the form of the **screened potential**;
- $H + \Phi$ scattering in the **strong-coupling regime**.

At variance with present analyses still largely based on weak-coupling techniques
 (Peskin 79, Bhanot Peskin 79):



The diagram illustrates the transition from a bare interaction to a screened interaction. On the left, two external lines representing gluons (gE_i^a and gE_j^a) interact via a bare coupling (indicated by a crossed line with a circle) to produce a virtual quarkonium state. This state then interacts with a background field Φ (represented by a wavy line). On the right, the final state consists of the same two gluons (gE_i^a and gE_j^a) interacting with a screened coupling (indicated by a crossed line with a dot), which is represented by a loop diagram involving the quarkonium state and the field Φ . The transition is indicated by a blue arrow.

$$= a^{ij} e^{-iT\epsilon^{\text{bind}}} \int_{-\infty}^{\infty} dt gE_j^a(t) gE_i^a(t)$$

The matching coefficient a^{ij} is:

$$a^{ij} = -\frac{V_A^2}{2N_c} \langle \Phi | r_j \frac{-i}{H_{\text{octet}} - \epsilon^{\text{bind}}} r_i | \Phi \rangle$$

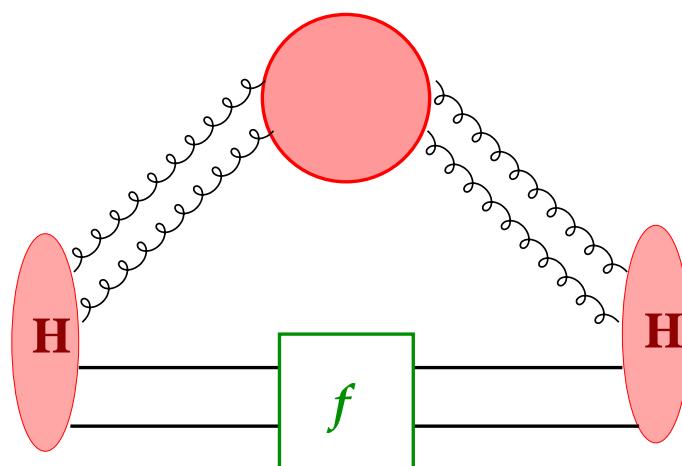
Inclusive Decays in NRQCD

$$\Gamma(H \rightarrow \text{LH}) = -2 \operatorname{Im} \langle H | \mathcal{H} | H \rangle$$

$$= \sum_n \frac{2 \operatorname{Im} f^{(n)}}{m^{d_n-4}} \langle H | \psi^\dagger K^{(n)} \chi \chi^\dagger K'^{(n)} \psi | H \rangle$$

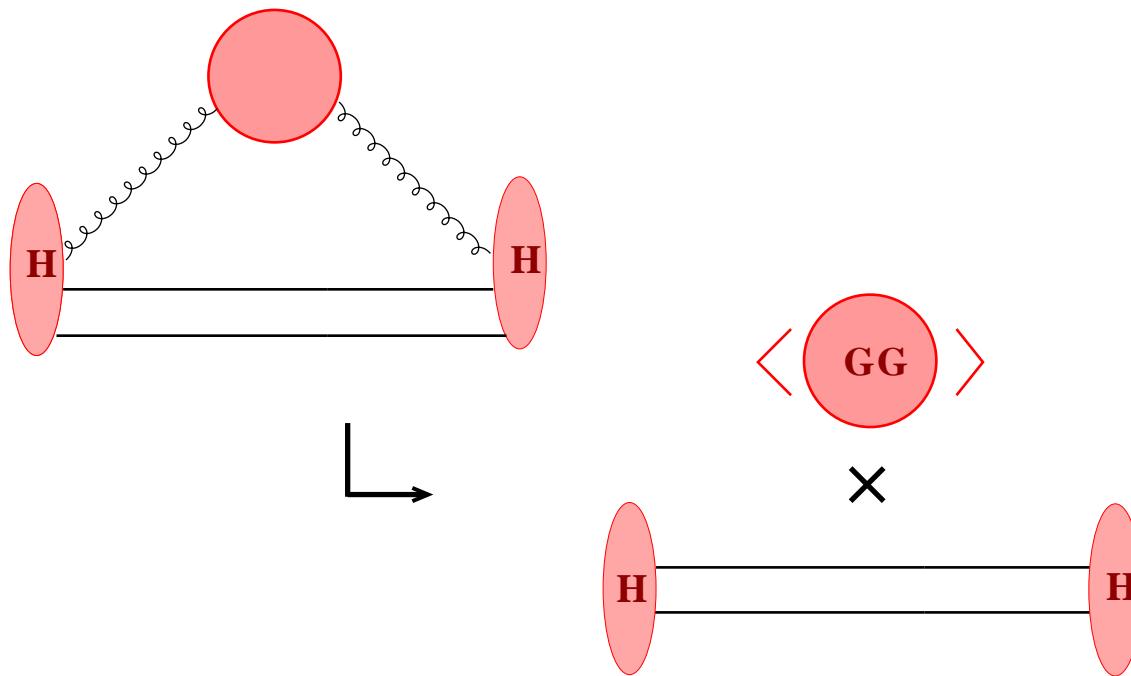
$$\Gamma(H \rightarrow \text{EM}) = \sum_n \frac{2 \operatorname{Im} f_{\text{em}}^{(n)}}{m^{d_n-4}} \langle H | \psi^\dagger K^{(n)} \chi | \text{vac} \rangle \langle \text{vac} | \chi^\dagger K'^{(n)} \psi | H \rangle$$

Bodwin et al 95



Inclusive Decays in pNRQCD

$$\langle H | \psi^\dagger K^{(n)} \chi \chi^\dagger K'^{(n)} \psi | H \rangle = |R(0)|^2 \times \int dt t^n \langle G(t) G(0) \rangle$$



P-wave decays at $\mathcal{O}(mv^5)$

- NRQCD

$$\Gamma(\chi_J \rightarrow \text{LH}) = 9 \text{ Im } f_1 \frac{|R'(0)|^2}{\pi m^4} + \frac{2 \text{ Im } f_8}{m^2} \langle \chi | O_8(^1S_0) | \chi \rangle$$

$$\Gamma(\chi_J \rightarrow \gamma\gamma) = 9 \text{ Im } f_{\gamma\gamma} \frac{|R'(0)|^2}{\pi m^4} \quad J = 0, 2$$

* *Bottomonium and charmonium P-wave decays depend on 6 non-perturbative parameters.*

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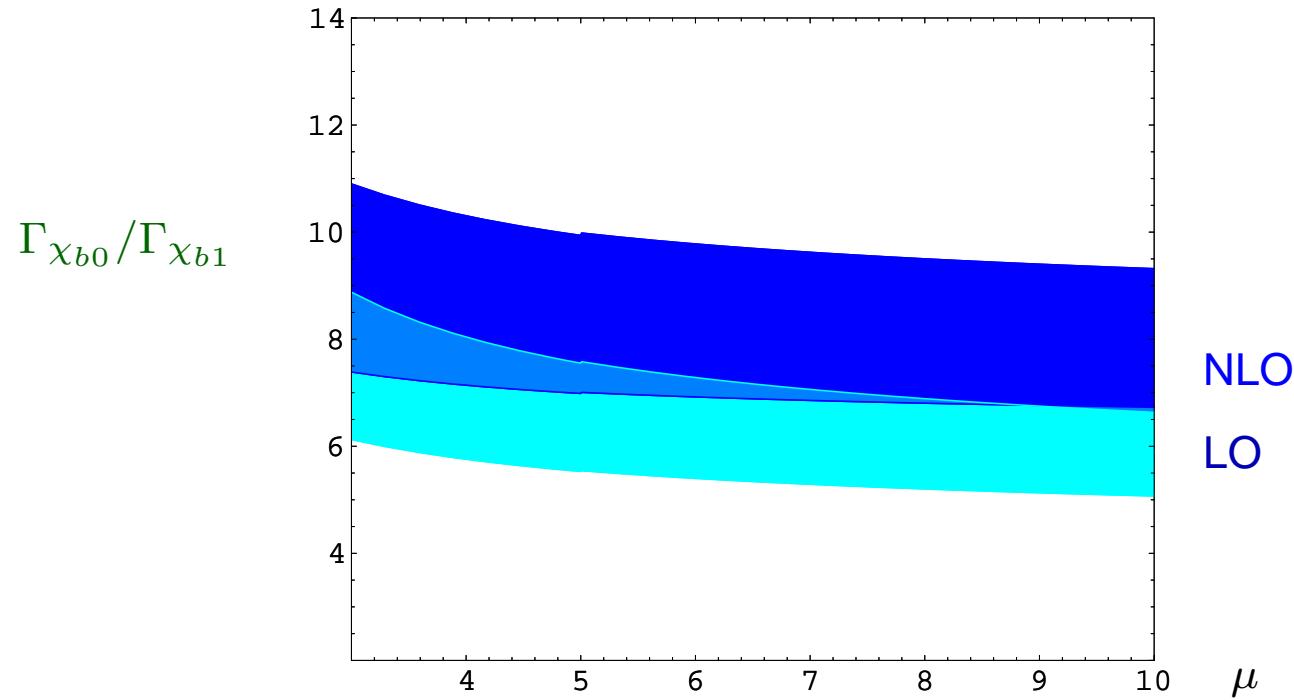
- pNRQCD

$$\langle \chi | O_8(^1S_0) | \chi \rangle = \frac{|R'(0)|^2}{18\pi m^2} \mathcal{E}; \quad \mathcal{E} \equiv \int_0^\infty dt t^3 \langle \text{Tr}(g\mathbf{E}(t) g\mathbf{E}(0)) \rangle$$

* *The quarkonium state dependence factorizes.*

* *Bottomonium and charmonium P-wave decays depend on 4 non-perturbative parameters.*

Bottomonium P -wave decays



$$\frac{\Gamma(\chi_{b0}(1P) \rightarrow \text{LH})}{\Gamma(\chi_{b1}(1P) \rightarrow \text{LH})} = \frac{\Gamma(\chi_{b0}(2P) \rightarrow \text{LH})}{\Gamma(\chi_{b1}(2P) \rightarrow \text{LH})} = 8.0 \pm 1.3$$

(Cleop II 02) = 19.3 ± 9.8

S-wave octet matrix elements

At leading order in the v and Λ_{QCD}/m expansion:

$$\langle V | O_8(^3S_1) | V \rangle = \langle P | O_8(^1S_0) | P \rangle = 3 \frac{|R(0)|^2}{2\pi} \left(-\frac{\mathcal{E}_3^{(2)}}{9m^2} \right)$$

$$\langle V | O_8(^1S_0) | V \rangle = \frac{\langle P | O_8(^3S_1) | P \rangle}{3} = 3 \frac{|R(0)|^2}{2\pi} \left(-\frac{c_F^2 \mathcal{B}_1}{18m^2} \right)$$

$$\frac{\langle V | O_8(^3P_J) | V \rangle}{2J+1} = \frac{\langle P | O_8(^1P_1) | P \rangle}{9} = 3 \frac{|R(0)|^2}{2\pi} \left(-\frac{\mathcal{E}_1}{54} \right)$$

$$\langle \chi | O_8(^1S_0) | \chi \rangle = \frac{1}{6} \frac{|R'(0)|^2}{\pi m^2} \mathcal{E}_3$$

$$\begin{aligned} \langle V | \mathcal{P}_1(^3S_1) | V \rangle &= \langle P | \mathcal{P}_1(^1S_0) | P \rangle = \langle V | \mathcal{P}_{\text{EM}}(^3S_1) | V \rangle \\ &= \langle P | \mathcal{P}_{\text{EM}}(^1S_0) | P \rangle = 3 \frac{|R(0)|^2}{2\pi} \left(m E_{n0}^{(0)} - \mathcal{E}_1 \right) \end{aligned}$$

S-wave decays

-

$$R_n^V \equiv \frac{\Gamma(V(nS) \rightarrow LH)}{\Gamma(V(nS) \rightarrow e^+e^-)} \quad R_n^P \equiv \frac{\Gamma(P(nS) \rightarrow LH)}{\Gamma(P(nS) \rightarrow \gamma\gamma)}$$

It is a prediction of pNRQCD that, for the states for which $\Lambda_{\text{QCD}} \gg mv^2$, the wave-function dependence drops out.

[Residual m dependence in $1/m$, $E_{n0}^{(0)}$ and $\text{Im } f$; residual n dependence in $E_{n0}^{(0)}$.]

Brambilla Eiras Pineda Soto Vairo 01

S-wave decays

•

$$\frac{R_n^V}{R_m^V} = 1 + \left(\frac{\text{Im } g_1(^3S_1)}{\text{Im } f_1(^3S_1)} - \frac{\text{Im } g_{ee}(^3S_1)}{\text{Im } f_{ee}(^3S_1)} \right) \frac{M_n - M_m}{m},$$
$$\frac{R_n^P}{R_m^P} = 1 + \left(\frac{\text{Im } g_1(^1S_0)}{\text{Im } f_1(^1S_0)} - \frac{\text{Im } g_{\gamma\gamma}(^1S_0)}{\text{Im } f_{\gamma\gamma}(^1S_0)} \right) \frac{M_n - M_m}{m}.$$

For $m_b = 5 \text{ GeV}$, $R_2^\Upsilon / R_3^\Upsilon \simeq 1.3$ [pdg $\simeq 1.4$]

$\text{Im } g_1(^1S_0) / \text{Im } f_1(^1S_0) - \text{Im } g_{\gamma\gamma}(^1S_0) / \text{Im } f_{\gamma\gamma}(^1S_0) \sim \alpha_s$
 \Rightarrow up to $\mathcal{O}(v^3)$, R_n^P is equal for all radial excitations.